Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 251-3, Fall 2005
Midterm 2
November 2nd, 2005, 8:30 – 9:20

Last Name (please print):	KEY	
First Name (please print):		
Student Number:		
Instructor:	D. Marinescu	P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table!	
Question	**Marks**
1	/2
2	/3
3	/3
4	/4
5	/4
6	/4
Total	**/20**

1. Match the function description with its contour map by placing the letter next to the description in the square box next to the graph.
   [1/2 mark each=2 marks]

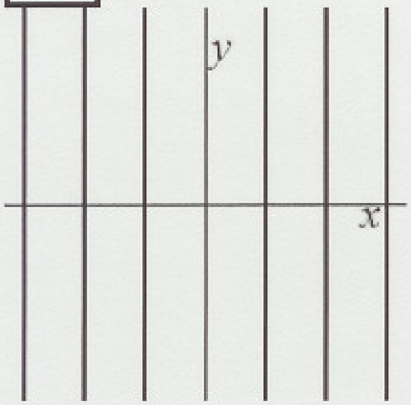
   a) A function whose graph is the shape of a cone.

   b) A function whose graph is the shape of a sphere.

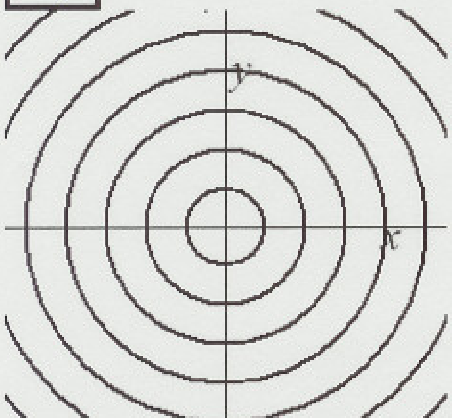
   c) A function whose graph is the shape of a Paraboloid.

   d) The function $z = x$ whose graph is the shape of a $z \cancel{=} x$ plane.

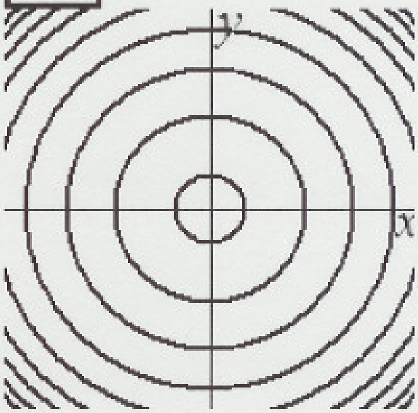
e) d



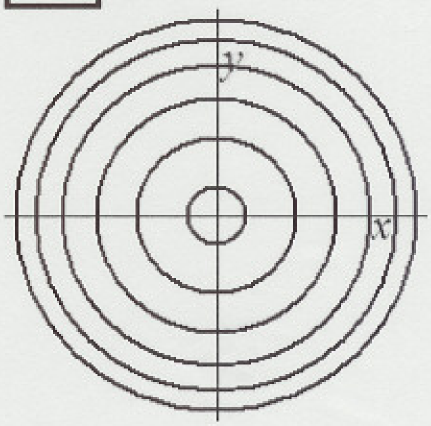
f) a



g) c



h) b



2. Find $\lim_{(x,y)\to(0,0)} \dfrac{5xyz}{x^2+y^2+z^2}$. **[3 marks]**

$$= \lim_{\rho\to 0} \frac{5(\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)(\rho\cos\phi)}{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta + \rho^2\cos^2\phi}$$

$$= \lim_{\rho\to 0} \frac{5\rho^3\sin^2\phi\cos\phi\cos\theta\sin\theta}{\rho^2\sin^2\phi + \rho^2\cos^2\phi}$$

$$= \lim_{\rho\to 0} \frac{5\rho\sin^2\phi\cos\phi\cos\theta\sin\theta}{1}$$

$= 0$ since $\sin\phi, \cos\phi, \cos\theta, \sin\theta$ are bounded

3. Find the maximum rate of change of f at $(1,-1,1)$ given that $f(x,y,z) = x^4y^3z^2$. **[3 marks]**

$$\nabla f = \langle 4x^3y^3z^2, 3x^4y^2z^2, 2x^4y^3z \rangle$$

$$\nabla f(1,-1,1) = \langle -4, 3, -2 \rangle$$

$$|\nabla f(1,-1,1)| = \sqrt{(-4)^2+3^2+(-2)^2} = \sqrt{29}$$

is the maximum rate of change at $(1,-1,1)$ of f.

4. Given $j=\frac{1}{k}+\sqrt{l}-\ln m$, $k=\sin n+\cos p$, $l=np^{+1}$, $m=e^{n+p-\pi}$, $n=q^2$, $p=\pi q$.

[4 marks]

a) Find ~~j~~, k., l, m, n, p for $q=0$.

$p=0$, $n=0$, $m=e^{-\pi}$, $l=1$, $k=1$

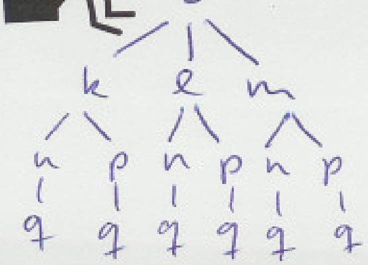
b) Find $\frac{dn}{dq}$ and $\frac{dp}{dq}$.

$\frac{dn}{dq}=2q$ $\qquad$ $\frac{dp}{dq}=\pi$

c) Find $\frac{\partial k}{\partial p}$. $= -\sin p$

$$\frac{\partial j}{\partial q}=\frac{\partial j}{\partial k}\cdot\frac{\partial k}{\partial n}\frac{dn}{dq}+\frac{\partial j}{\partial k}\frac{\partial k}{\partial p}\frac{dp}{dq}+\frac{\partial j}{\partial l}\frac{\partial l}{\partial n}\frac{dn}{dq}$$
$$+\frac{\partial j}{\partial l}\frac{\partial l}{\partial p}\frac{dp}{dq}+\frac{\partial j}{\partial m}\frac{\partial m}{\partial n}\frac{dn}{dq}+\frac{\partial j}{\partial m}\frac{\partial m}{\partial p}\frac{dp}{dq}$$

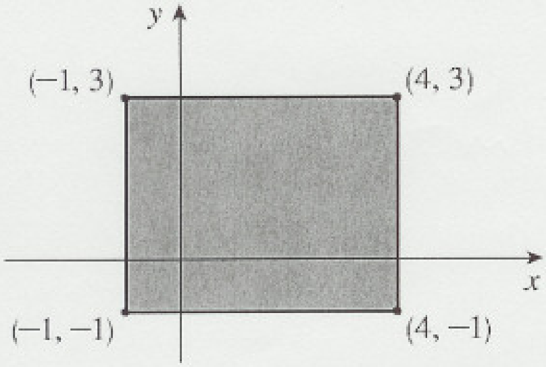
d) Find $\left.\frac{\partial j}{\partial q}\right|_{q=0}$.



$$=\left(-\frac{1}{k^2}\cos n+\frac{p}{2\sqrt{l}}-\frac{e^{n+p-\pi}}{m}\right)2q$$
$$+\left(+\frac{1}{k^2}\sin p+\frac{n}{2\sqrt{l}}-\frac{e^{n+p-\pi}}{m}\right)\pi$$

$$\left.\frac{\partial j}{\partial q}\right|_{q=0}=-\pi$$

5. Let $f(x,y)=e^{-(x^2+y^2)}$. Find the maximum and minimum values of f on the rectangle shown below. Justify your answer. **[4 marks]**

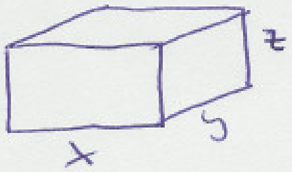


- f is constant on circles $x^2+y^2=r^2$
- f is decreasing as the radius of the circle increases

So, maximum occurs at $(0,0)$ and its value is $f(0,0)=1$, and minimum occurs at $(4,3)$ and its value is $f(4,3)=e^{-25}$.

Use Lagrange multipliers to

6. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is 24. **[4 marks]**



$$V(x,y,z) = xyz$$

$$4(x+y+z) = 24$$

$$\Rightarrow x+y+z = 6$$

$$\left\{\begin{array}{ll} x+y+z = 6 & (3) \\ yz = \lambda & \\ xz = \lambda & \\ xy = \lambda & \end{array}\right.$$

$$\Rightarrow \underbrace{yz = xz}_{y = x \;(1)} = xy$$

$$xz = xy \Rightarrow z = y \quad (2)$$

(1) and (2) into (3):

$$y + y + y = 6$$

$$y = 2, \quad x = 2, \quad z = 2$$

The dimensions are $2 \times 2 \times 2$, a square.