Simon Fraser University

MATH 251 - Summer 2005

Midterm 1

June 1, 2005, 8:30 – 9:20 am

Last Name (please print): ______________________________

First Name (please print): ______________________________

Student Number: ______________________________

Signature: ______________________________

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 6 pages with a total of 6 questions. Once the exam begins please check to make sure your exam booklet is complete.
4. Only complete well-organized solution will receive full credit
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Question	Marks
1	/8
2	/6
3	/4
4	/4
5	/10
6	/8
Total	**/40**

1.

a) Suppose that M is the midpoint of the segment AB in space and that P is another point. Show that $\overrightarrow{PM} = \frac{1}{2}(\overrightarrow{PA} + \overrightarrow{PB})$. **[4 marks]**

b) Suppose that $\vec{a}$ and $\vec{b}$ are two unit vectors. Show that $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$. **[4 marks]**

2. Find the equation of the plane that contains the two parallel lines

$$L_1:\begin{cases} x=1+t \\ y=-1+2t \\ z=2+3t \end{cases} \text{ and } L_2:\begin{cases} x=2+2s \\ y=4s \\ z=1+6s \end{cases}.$$ **[6 marks]**

3. Determine whether or not the four points $A(2,0,-3)$, $B(0,5,4)$, $C(1,1,-1)$ and $D(5,-12,-18)$ are coplanar. **[4 marks]**

4. A particle moves along a curve C with velocity given by $\vec{v}(t) = < 2t, e^t, \sin(\pi t) >$. At time $t=1$, the particle passes through the point $P(1,0,1/\pi)$. Determine the position vector $\vec{r}(t)$ at time t. **[4 marks]**

5. A space curve is given by the vector function $\vec{r}(t) = < 4t, t^2, 2t^2 >$.

a) Find the equation of the line that is tangent to its position vector and that is parallel to the line: $\frac{x-1}{2} = y = \frac{z+1}{2}$. **[6 marks]**

b) Find the curvature of curve κ for $t \geq 0$.

(Hint: $\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{1}{v}\left|\frac{d\vec{T}}{dt}\right| = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$). **[4 marks]**

6.

a) Convert the equation $x^2 + y^2 = 4$ both to cylindrical and to spherical coordinates. **[4 marks]**

b) Find the domain of the function $z = \sqrt{25 - x^2 - y^2}$. Sketch its level curves at $z = 0, 3, 4$ in the *xy*-plane. **[4 marks]**