Simon Fraser University

MATH 251
Summer 2004
First Midterm Examination

Instructor: A. Belshaw Date: June 2, 2004

Student: _____

Student number: _____

Signature: _____

Instructions

1. Fill in the information above.
2. Please do not open the examination booklet until you are told to do so.
3. Do all your work in this test booklet. Show all your work. Use the backs of the pages if necessary.
4. No books, no notes, and no calculators.

1	2	3	4	Total

1. **[12 marks]** Let four points be given by $A = (0, 0, 0)$, $B = (1, 1, 1)$, $C = (-1, 1, 1)$, $D = (1, 0, 3)$. Find the following:

   (a) **[2]** the volume of the parallelopiped formed by $\vec{AB}$, $\vec{AC}$, and $\vec{AD}$;

   (b) **[2]** the angle between $\vec{AB}$, and $\vec{AD}$;

   (c) **[4]** the equation of the plane containing $\vec{BD}$ and $\vec{BC}$;

   (d) **[4]** the angle between the plane in (c) and the plane containing $\vec{BD}$ and $\vec{AD}$

2. [**15 marks**] Given the parametric equation of the curve $\mathbf{r}(t) = \mathbf{i}\cos t + \mathbf{j}4\sin t + \mathbf{k}2t$, find:

   (a) [**1**] $\mathbf{r}(\pi)$;

   (b) [**2**] the velocity at $\mathbf{r}(\pi)$;

   (c) [**2**] the equation of the tangent line to the curve at $\mathbf{r}(\pi)$; ;

   (d) [**2**] the curvature of the curve at $\mathbf{r}(\pi)$;

   (e) [**2**] the centre of the osculating circle at $\mathbf{r}(\pi)$;

(f) **[4]** the normal and tangential components of acceleration at $\mathbf{r}(\pi)$;

(g) **[2]** the arc distance from $\mathbf{r}(\pi)$ to $\mathbf{r}(2\pi)$.

3. [**11 marks**]

(a) [**5**] Find the cylindrical coordinate equation for the ellipsoid given by

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 = 4.$$

(b) [**6**] Find the spherical coordinate equation for

$$x^2 + y^2 - z^2 = 1.$$

4. **[12 marks]** Given $f(x, y) = \sqrt{x^2 + y^2 - 1}$,

   (a) **[6]** sketch level curves for $f(x, y) = 0, 1, 2, 3$;

   (b) **[6]** sketch the graph of the surface $z = f(x, y)$, showing traces parallel to the xy plane and parallel to the yz plane.