MATH 251-3, Spring 2006 **Simon Fraser University**

Midterm 1 7 February 2006, 5:30–6:20pm

Instructor: Ralf Wittenberg

Last Name:	______________________
First Name:	______________________
Student Number:	______________________
Signature:	______________________

Instructions

1. PLEASE DO NOT OPEN THIS BOOKLET UNTIL INVITED TO DO SO.
2. Write your last name, first name(s) and student number in the box above in block letters, and sign your name in the space provided.
3. This exam contains 5 questions on 5 pages (after this title page). Once the exam begins please check to make sure your exam is complete.
4. The total time available is 50 minutes, and there are 50 points, so allow about a minute per point; for example, you should aim to spend about 10 minutes on a 10-point question. Attempt all problems!
5. This is a closed book exam. Only non-programmable scientific calculators are allowed.
6. Use the reverse side of the previous page if you need more room for your answer, and clearly indicate where the solution continues.
7. Show all your work, and explain your answers clearly.
8. Good luck!

Question	Maximum	Score
1	18	
2	4	
3	10	
4	10	
5	8	
Total	50	

1. Given the four points

$$A(1,2,1), \quad B(2,0,1), \quad C(-1,2,0), \quad \text{and} \quad D(3,3,-1) \ :$$

   (a) [2 points]
   Compute $\overrightarrow{AB} \times \overrightarrow{AC}$.

   (b) [2 points]
   Find the area of the triangle ABC.

   (c) [4 points]
   Find the angle θ between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   (d) [6 points]
   Find an equation of the plane that passes through A and B and is parallel to the line through C and D.

Q.1, continued from previous page ...

(e) [4 points]
Find the distance from C to the plane determined in part (d).

2. [4 points]
The angular momentum $\mathbf{L}(t)$ and the torque $\tau(t)$ of a moving particle of mass m with position vector $\mathbf{r}(t)$ are defined to be

$$\mathbf{L}(t) = m\,\mathbf{r}(t) \times \mathbf{v}(t), \qquad \tau(t) = m\,\mathbf{r}(t) \times \mathbf{a}(t),$$

where $\mathbf{v} = \mathbf{r}'$, $\mathbf{a} = \mathbf{r}''$. Prove that $\mathbf{L}'(t) = \tau(t)$.

3. The *velocity* of a particle moving along a helical path is given by the vector

$$\mathbf{v}(t) = 3\,\mathbf{i} + 4\sin t\,\mathbf{j} + 4\cos t\,\mathbf{k}\,, \quad t \geq 0.$$

(a) [3 points]
At time $t = 0$, the particle passes through the point $P(1, 0, 0)$. Find the position vector $\mathbf{r}(t)$ for the particle.

(b) [3 points]
Find the distance travelled (arc length) along the helical curve from $t = 0$ to $t = 2\pi$.

(c) [4 points]
Find the tangent vector $\mathbf{T}$ and principal normal vector $\mathbf{N}$ to the helical curve.

4. A particle moves in space with parametric equations

$$x = t, \qquad y = t^2, \qquad z = \tfrac{4}{3}t^{3/2}, \qquad t > 0.$$

   (a) [4 points]
   Find the velocity and acceleration vectors, and the speed of the particle as a function of t.

   (b) [6 points]
   Determine the curvature κ and radius of curvature of the curve traced out by the particle at time $t = 1$.

5. Consider the surface described by the equation

$$x^2 + y^2 - z^2 = 4.$$

   (a) [4 points]
   Briefly describe the traces (cross-sections) of the surface in horizontal planes of the form $z = z_0$, *and* the traces (cross-sections) of the surface in vertical planes of the form $x = x_0$. (You may wish to include a rough sketch in your description.)

   (b) [4 points]
   Write the equation for the surface in both cylindrical and spherical coordinates.