MATH 251
Simon Fraser University
Department of Mathematics
Burnaby Campus
Midterm Exam No. 1
Feb. 9, 2005
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DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED. Once the test begins, please check that all pages are intact. Answer all five questions, clearly showing your working. Each question is worth 8 marks. If you run out of space use the extra space on the back of each page, CLEARLY INDICATING WHERE THE SOLUTION CONTINUES AND THE NUMBER OF THE QUESTION THAT YOU ARE ANSWERING. Ordinary scientific calculators are allowed. The duration of the test is 40 minutes.

Name:

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

TOTAL:

1a) (4 marks) Consider the plane surface $x - 2y + z = 3$. Find equations for the intersection of this surface with each of the coordinate planes.

1b) (4 marks) Consider the **surface of revolution** generated by revolving the curve $y = x^2$ around the y-axis. By examining the evolution of a point $P(x_0, y_0)$ on the curve, determine the equation for the surface of revolution.

2a) (1 mark) Consider the curve C given by the equation $y = 2x^2$. Find a parametric representation of C and hence write down the position vector that generates C.

2b) (2 marks) Evaluate the unit tangent vector to C at the point $(x, y) = (1, 2)$.

2c) (3 marks) Evaluate the principle unit normal to C at the point $(x, y) = (1, 2)$.

2d) (2 marks) Evaluate the curvature of C at the point $(x, y) = (1, 2)$.

3a) Consider the vectors $\vec{u} = 2i - j + 4k$, $\vec{v} = i - 3j - 2k$ and $\vec{w} = i + 6j + k$.

(i) (1 mark) Are $\vec{u}$ and $\vec{w}$ perpendicular or parallel? Explain your answer.

(ii) (3 marks) Find **two** vectors which are perpendicular to **both** $\vec{u}$ and $\vec{v}$.

3b) (4 marks) Given the vectors $\vec{v} = (v_1, v_2, v_3)$, $\vec{u} = (u_1, u_2, u_3)$ and $\vec{w} = (w_1, w_2, w_3)$ show that:

$$\vec{v} \cdot (\vec{u} + \vec{w}) = \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{w}$$

4a) (4 marks) Find both the cylindrical and spherical coordinates of the point $(0, 0, -1)$, given in rectangular Cartesian coordinates. **Note:** Rectangular coordinates are (x, y, z), cylindrical are (r, θ, z) and spherical are (ρ, ϕ, θ).

4b) (4 marks) Convert the equation $2x + y - z = 4$ into both cylindrical and spherical coordinates.

5a) (3 marks) Given the function $f(x, y) = \sqrt{1 - x^2 - 2y^2}$ determine its **largest possible domain**. Sketch the domain in the xy plane.

5b) Let $f(x, y) = x^2 + 2y^2$.

(i) (3 marks) Describe and sketch some typical level curves of $f(x, y)$.

(ii) (2 marks) Using your previous answer sketch the graph of $f(x, y)$, indicating some typical contour curves.