Simon Fraser University

MATH 251 - Summer 2005

Final Exam
Aug 5, 2005, 8:30 – 11:30 am

Last Name (please print): ________________________________

First Name (please print): ________________________________

Student Number: ________________________________

Signature: ________________________________

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 15 pages with a total of 11 questions. Once the exam begins please check to make sure your exam booklet is complete.
4. Only complete well-organized solution will receive full credit
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific calculators are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and a scientific calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

Question	Marks
1	/6
2	/12
3	/11
4	/6
5	/7
6	/10
7	/15
8	/8
9	/8
10	/9
11	/8
Total	**/100**

1. Suppose that a moving point has initial position vector $\vec{r}(0) = 2\vec{i}$, and velocity vector $\vec{v}(t) = (1-2t)\vec{i} + (3t^2 - 1)\vec{j}$. Find its position vector $\vec{r}(t)$ and acceleration vector $\vec{a}(t)$. **[6 marks]**

2. Let C be a curve defined by the parametric equations:

$$x = \cos t\,,\ \ y = \sin t\,,\ \ z = \sqrt{2}t\,,\ \ 0 \le t \le 2\pi\ .$$

a) Find the arc length of the curve C. **[6 marks]**

b) Find the equation of the line that is tangent to the curve C and that is parallel to the line: $x = -t + 1, y = t + 2, z = 2t + 3$. **[6 marks]**

3.

a) Determine all values of a such that the vector $\vec{v} = < a^2, -2a, -1 >$ lies in the plane tangent to the surface $z = e^x / y$ at the point $(0,1,1)$. **[6 marks]**

b) For a differentiable function *f*, given $D_{\vec{u}} f(0,0) = 2\sqrt{2}$ and $D_{\vec{v}} f(0,0) = -\sqrt{2}$, where $\vec{u} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ and $\vec{v} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$, find $f_x(0,0)$ and $f_y(0,0)$.

[5 marks]

4. Find the limit $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$, if it exists. **[6 marks]**

5. Find the linear approximation of $f(x, y) = 4\sqrt{\sqrt{x} + \sqrt{y} + 1}$ at the point $(1, 4)$. Use this to estimate $4\sqrt{\sqrt{1.2} + \sqrt{3.9} + 1}$. **[7 marks]**

6. Find the absolute maximum and minimum for the function $f(x, y) = 2xy$ on the region $R = \{(x, y) \mid x^2 + y^2 \le 2\}$. **[10 marks]**

7.

a) Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dxdy$ by first reversing the order of integration.

[7 marks]

b) Find the volume of the solid region that lies below the surface $z = \dfrac{xy}{1+x^2y^2}$ and above the plane region *R*. *R* is bounded by the graphs of $xy = 1$, $xy = 4$, $x = 1$ and $x = 4$. (Hint: Let $u = x$, $v = xy$.) **[8 marks]**

8. Find the surface area S of the portion of the sphere $x^2 + y^2 + z^2 = 25$ that lies above the plane $z = 4$. **[8 marks]**

9. Find the moment of inertia $I_z = \iiint_T (x^2 + y^2)\delta(x, y, z)\, dV$ of the solid T with $\delta(x, y, z) = z$, where the solid T lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$ in the first octant. **[8 marks]**

10.

a) Given $f(x,y,z) = \tan x + z^2 \ln y$, find div (∇f) and curl (∇f). **[5 marks]**

b) Determine if the vector field $\vec{F}(x,y) = (x^2 + ye^{xy})\vec{i} + (y^2 + xe^{xy})\vec{j}$ is conservative. **[4 marks]**

11. Evaluate the line integral $\int_C xydx + (x^2 + y^2)dy$, where C is described by the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ and the line segment $y = 1$ from $(1,1)$ to $(2,1)$. **[8 marks]**