Simon Fraser University

MATH 251
Summer 2004
Final Examination

Instructor: A. Belshaw   Date: August 4, 2004

Name: ______________________________

Student number: ______________________________

Signature: ______________________________

Instructions

1. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.
2. Fill in the information above.
3. Have your student ID card showing on the desk.
4. This booklet contains 11 printed pages in addition to this cover page.
5. Do all your work in this test booklet. Show all your work. Use the back of the **previous page** if necessary.
6. No book, paper, or device should be within reach.
7. Students observed writing anything after the call to stop writing will be subject to summary penalties.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
6	4	8	8	6	5	8	8	6	6	8	5	6	8	8	100

[6] 1. Calculate the length of the curve parametrized by

$$\mathbf{r}(t) = \langle t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}, \frac{t^2}{2}\rangle$$

from $t = 0$ to $t = 2$.

[4] 2. Find the equation of the surface generated by revolving the curve $y = e^{-z}$ around the z-axis.

[8] 3. Let $\mu(t) = \mathbf{r}(t) \times \mathbf{v}(t)$, and $\tau(t) = \mathbf{r}(t) \times \mathbf{a}(t)$, where $\mathbf{r}$, $\mathbf{v}$ and $\mathbf{a}$ are the position, velocity, and acceleration of a particle moving through space as functions of time. Prove that

$$\mu'(t) = \tau(t).$$

[8] 4. Using the definition of curvature and a general parametric equation of a line in three dimensions, prove that the curvature of a straight line in space is zero.

[6] 5. Given $f(x,y,z) = \dfrac{xyz}{x^2+y^2+z^2}$,
prove, using spherical coordinates, whether or not

$$\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$$

exists. If the limit exists, find it.

[5] 6. Use a double integral to find the volume bounded above by the paraboloid $x^2 + y^2 + \dfrac{z}{2} = 1$ and below by the xy-plane.

[8] 7. Given $\phi(x, y, z) = \dfrac{1}{r}$, where $r = \sqrt{x^2 + y^2 + z^2}$, prove that

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \ .$$

[8] 8. Find the maximum value of $f(x, y, z) = xyz$, on the intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the plane $z = 1/2$, using the method of Lagrange multipliers.

[6] 9. Use triple integration to find the moment of inertia around the z-axis of the solid bounded above by $z = 3$ and below by the paraboloid $z = 2x^2 + 2y^2$. Assume density $\delta \equiv 1$.

[6] 10. Find the surface area of the surface parametrized by $\mathbf{r}(u, v) = \langle \sin u, \cos u, v \rangle$ for $0 \leq u \leq 2\pi$ and $1 \leq v \leq 2$.

[8] 11. Use the transformation given by $x = 2r\cos\theta$ and $y = r\sin\theta$ to find the volume of the solid bounded by the xy-plane, the paraboloid $z = x^2 + y^2$, and the elliptic cylinder $\dfrac{x^2}{4} + y^2 = 1$. (Use the fact that $\displaystyle\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$.)

[5] 12. Prove that curl(grad$f$) $= \mathbf{0}$, if the real-valued function $f(x, y, z)$ has continuous second-order partial derivatives.

[6] 13. Use the vector form of Green's theorem to calculate the flux of the field $\mathbf{F}(x, y) = \langle x^2, y^2 \rangle$ across the circle $x^2 + y^2 = 4$.

[8] 14. Find the centroid of a half-turn of wire of uniform density given by $\mathbf{r}(t) = \langle 5\cos t, 5\sin t, t\rangle$, letting t run from 0 to π. Show either that the centroid is on the wire or that it is not on the wire.

[8] 15. Show that the vector field $\mathbf{F}(x, y) = \langle y \cos x + \cos y, \sin x - x \sin y \rangle$ is conservative. Then find a potential function for $\mathbf{F}$ by the method of integration.