Simon Fraser University
Department of Mathematics
Burnaby and Surrey Campus

MATH 251-3, Fall 2005
Final Examination
December 7th, 2005

Last Name (please print): ______________________________

First Name (please print): ______________________________

SFU email ID: ______________________________

Instructor: D. Marinescu P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 15 pages with a total of 13 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table!

Question	**Marks**
1	**/6**
2	**/8**
3	**/5**
4	**/10**
5	**/5**
6	**/4**
7	**/8**
8	**/10**
9	**/10**
10	**/6**
11	**/10**
12	**/10**
13	**/8**
Total	**/100**

1. Consider the points $A(2,3,2)$, $B(4,1,0)$, $C(-1,2,0)$, and $D(5,4,-2)$.

   a) Find an equation of the plane that passes through *A* and *B* and is parallel to the line through *C* and *D*. **[4 marks]**

   b) Find the distance from *C* to the plane determined in part a). **[2 marks]**

2. A particle moves in space with parametric equations $x = t, y = t^2, z = \frac{4}{3}t^{3/2}$.

a) Find the velocity and acceleration vectors. Find the speed. **[3 marks]**

b) The acceleration vector can be decomposed into its tangential and normal components as follows: $\mathbf{a}(t) = \frac{dv}{dt}\,\mathbf{T} + \kappa v^2\,\mathbf{N}$, where v is the speed, **T** is the unit tangent vector, κ is the curvature**,** and **N** is the principal unit normal vector. Determine κ and **N** at time $t = 1$. **[5 marks]**

3. Determine if the function h given below is continuous on its domain. **[5 marks]**

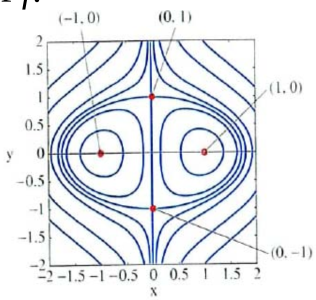
$$h(x,y,z)=\begin{cases}\dfrac{\sin xyz}{xyz}, & xyz \neq 0\\ 1, & xyz = 0\end{cases}$$

4. An open topped rectangular box is to have a total surface area of 300 square centimeters. Use Lagrange multipliers to find the dimensions of the box that maximizes its volume. **[10 marks]**

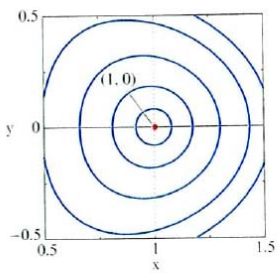
5. A pyramid is bounded by the three coordinate planes and by the plane tangent to the surface $xyz = 1$ at the point (a, b, c) in the first octant. Find the volume of this pyramid. Hint: The volume of a pyramid is one-third the product of the area of its base and its height. **[5 marks]**

6. Consider the function $f(x,y) = 3x - x^3 - 3xy^2$. Use the level curves below to identify the critical points of the function *f* on its domain $\mathbb{R}^2$ and make a reasonable suggestion whether the critical point will yield a local maximum, local minimum or saddle. **[4 marks]**

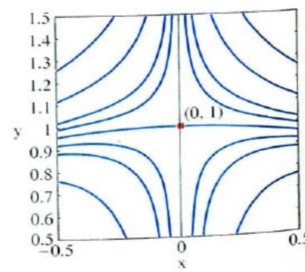
i) Level curves of *f*.



ii) Level curves of *f* near $(1,0)$.



iii) Level curves of *f* near $(0,1)$.



7. Suppose that the temperature, in degrees Celsius, at the point (x, y, z) in space is given by the formula $\boldsymbol{W} = 100 - \boldsymbol{x}^2 - \boldsymbol{y}^2 - \boldsymbol{z}^2$. The units for distance are meters.

a) Find the rate of change of the temperature at the point $P(3, -4, 5)$ in the direction of the vector $\vec{v} = 3\vec{i} - 4\vec{j} + 12\vec{k}$. **[4 marks]**

b) In what direction does W increase most rapidly at P? What is the value of the maximal directional derivative at P? **[4 marks]**

8. Rewrite the following integrals but do **not** evaluate.

a) Rewrite $\int_0^{2\pi}\int_0^1 r^2\, dr d\theta$ in rectangular coordinates. **[6 marks]**

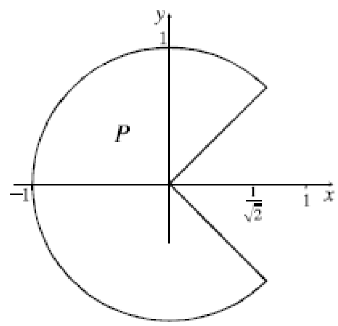
b) Consider $\int_0^1\int_{y^3}^{\sqrt{y}}\int_0^{xy} dzdxdy$ representing the solid S. **[4 marks]**

i) Let R be the projection of S in the xy-plane. Draw the region R.

ii) Rewrite $\int_0^1\int_{y^3}^{\sqrt{y}}\int_0^{xy} dzdxdy$ as $\iiint_S dzdydx$.

9. Evaluate the following integrals.

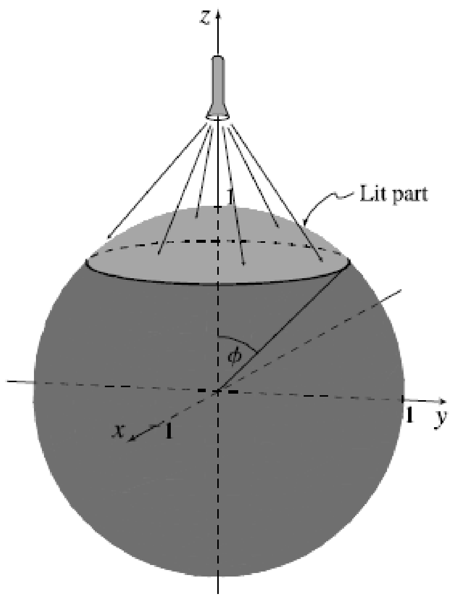
a) $\iint\limits_{P} x\,dA$, where the region P is graphed below. **[4 marks]**



b) $\iiint\limits_{S} ze^{\left(x^2+y^2+z^2\right)^2}\,dV$, where $S=\left\{(x,y,z) \text{ in } 1^{\text{st}} \text{ octant} / 4 \le x^2+y^2+z^2 \le 25\right\}$.

[6 marks]

10. A light on the z-axis, pointed at the origin, shines on the sphere $\rho = 1$ such that ¼ of the total surface area is lit. See the diagram below. What is the angle ϕ? Recall that the surface area of a sphere with radius r is $4\pi r^2$.
[6 marks]



11. Draw the image R under the transformations T described below of the unit square $S = \{(u,v)/0 \le u \le 1, 0 \le v \le 1\}$ and determine the Jacobian. **[5 marks each = 10 marks]**

a) $x = 2u + v, y = u + 2v$;

b) $x = v\cos 2\pi u, y = v\sin 2\pi u$.

12. Consider the integral $\int_C (2xy - 2y^2)\,dx + (x^2 - 4xy)\,dy$.

a) Is the integral path independent? **[2 marks]**

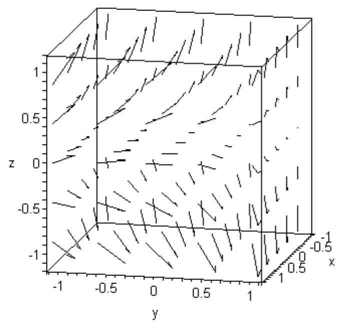
b) Find a potential function that evaluates the integral. **[4 marks]**

c) Find the value of the integral where C is the path from $(0,0)$ to $\left(\frac{5\pi}{2},1\right)$ along the curve $y = \sin x$. **[4 marks]**

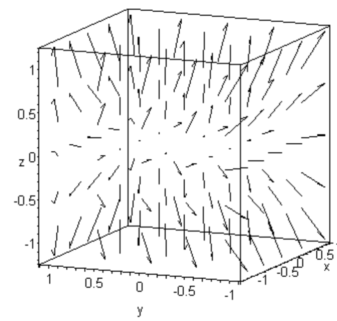
13. Consider the vector field $F(x, y, z) = 5e^{x} \sin y\vec{i} + 3e^{x} \cos y\vec{j} + 8z\vec{k}$.

a) Which plot given below matches *F*? **[2 marks]**

i)



ii)



b) Calculate the divergence *F*. **[3 marks]**

c) Calculate the curl of *F*. **[3 marks]**