MATH 251 Test 2, (R. Pyke) November 10, 2006

Time allotted: 50 minutes.
Total Marks: 62. Marks for each question are indicated by [ ].

(1) [10] Reparametrize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

$$\mathbf{r}(t) \;=\; 8\sin t\,\mathbf{i} \;+\; t\,\mathbf{j} \;+\; 8\cos t\,\mathbf{k}$$

(2) [8] Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$$

(3) [10] Find the indicated partial derivatives.

$$f(r,s,t) = r\ln(rs^2t^3); \qquad f_r, \quad f_{ss}, \quad f_{str}$$

(4) [8] Find the linearization of $z = e^{x^2-y^2}$ at the point $(1, -1, 1)$

(5) [8] Use the Chain Rule to find the partial derivatives $\partial z/\partial t$ and $\partial z/\partial s$;

$$z = xe^{y-z^2} \quad \text{where} \quad x = 2st, \quad y = s+t, \quad z = 2t - s$$

(6) [8] Find the directional derivative $\mathbf{D_u}f$ of $f(r,s,\theta) = \dfrac{e^{-r}\sin\theta}{s}$ at the point $(1,0,\pi/3)$ where $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

(7) [10] Find the local maximum and minimum values and saddle point(s) of the function

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$$

in the half-plane $x \geq -2$.