MATH 251 Test 1, (R. Pyke) October 4, 2006

Time allotted: 50 minutes.
Total Marks: 65. Marks for each question are indicated by [ ].

(1) [10] **(a)** Find **two** non-parallel unit vectors that are orthogonal to $\mathbf{v} = \langle 3, -1, 2 \rangle$

(b) Find a unit vector that is orthogonal to $\mathbf{v} = \langle 3, -1, 2 \rangle$ <u>and</u> $\mathbf{v} + \mathbf{u}$ where $\mathbf{u} = \langle -1, 4, 1 \rangle$.

(c) Let $\mathbf{w}$ and $\mathbf{z}$ be two unit vectors.

(i) Sketch $\mathbf{w} + \mathbf{z}$ and $\mathbf{w} - \mathbf{z}$.

(ii) Show that $\mathbf{w} + \mathbf{z}$ is orthogonal to $\mathbf{w} - \mathbf{z}$.

(2) [10] Use the vector projection $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$ to find the distance from the point $P(1,2,2)$ to the line $\dfrac{x-1}{2} = \dfrac{-y-4}{3} = \dfrac{z}{2}$. Begin by making a sketch of the line, the points $P(1,2,2)$, $Q(1,-4,0)$, and the vector $\vec{QP}$.

(3) [10] Prove the identity $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors in $\mathbf{R}^3$.

(4) [10] Find the equation of the plane that passes through the point $(1,-1,1)$ and contains the line $x=1+t,\ 2y=3-t,\ z=1+3t.$

(5) [10] Find the parametric equation of the line of intersection of the planes

$$x - 3y + z = 1, \qquad 2x + 3y - 4z = 2$$

(6) [10] Find the traces of the given surface in the planes $x = k,\ y = k,\ z = k$. Then sketch the surface.

$$4y = 2x^2 + z^2 - 1$$

(7) [5] Identify the surface given by $\rho \sin \phi = 4$.

(8) [10] Let the curve $\mathcal{C}$ be defined by $\mathbf{r}(t) = \langle t - t^2, \sin(\pi t), e^{2t} \rangle$. Find the parametric equations of the tangent line to the curve at the point $\mathbf{r}(2)$.