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Simon Fraser University
Department of Mathematics
Midterm Examination 2
MATH 232
14 November 2005 11:30–12:20

Question	Score	Maximum
1		8
2		8
3		8
4		4
5		12
Total		40

- Please ensure that you sign your exam above to certify your identity. Unsigned exams will not be marked.
- The duration of this exam is 50 minutes.
- DO NOT OPEN this test booklet until told to do so.
- Please check that you have all 6 pages of the exam.
- Do ALL your work in this test booklet. You may use the backside of each page for scrap work.
- The value of each question is shown on the left margins.

1. Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 4 & -1 & 3 & 4 & 0 \\ 1 & 3 & 4 & 1 & 0 \\ 2 & 3 & 4 & 4 & 0 \end{bmatrix}$ and suppose a row echelon form of A is

$$B = \begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 & 0 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

[4] (a) Determine a basis for Row(A).

[4] (b) Determine a basis for Row(A^t) which consists of rows of A^t.

2. Let $V = \mathbb{P}_2$. Consider $\mathcal{B} = \{1, 1+t, 1+t+t^2\}$ and $\mathcal{C} = \{1+t+t^2, 1+t, 2\}$ (where the vectors are ordered as given).

[4] (a) Show that $\mathcal{B}$ is a basis for V.

[4] (b) Find the change of basis matrix $P_{\mathcal{B} \leftarrow \mathcal{C}}$.

3. Let

$$A(\lambda) = \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & 1 & -\lambda \end{bmatrix}.$$

[4] (a) Show that the determinant of $A(\lambda)$ is $\lambda^2(\lambda^2 - 1)$.

[4] (b) Determine a basis for Nul$A(-1)$.

[2] **4.** (a) Using the answers from Question 3, determine the characteristic polynomial of

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

[2] (b) Using the answers from Question 3, determine a basis for the eigenspace of B corresponding to the eigenvalue -1.

5. Let V be a vector space over $\mathbb{R}$ with basis $\{\mathbf{b}_1, \mathbf{b}_2\}$. Let $T : V \to \mathbb{R}^2$ and $S : V \to \mathbb{R}^2$ be linear transformations.

[4] (a) Show that $R(\mathbf{x}) = S(\mathbf{x}) + 2T(\mathbf{x})$ is a linear transformation.

[4] (b) Suppose that $T(\mathbf{b}_1) = \mathbf{e}_1$, $T(\mathbf{b}_2) = \mathbf{e}_2$, $S(\mathbf{b}_1) = \mathbf{e}_2$, $S(\mathbf{b}_2) = \mathbf{e}_1$. Show that if $\alpha, \beta \in \mathbb{R}$ are such that $\alpha T(\mathbf{x}) + \beta S(\mathbf{x}) = \mathbf{0}$ for all $\mathbf{x} \in V$, then $\alpha = \beta = 0$.

[4] (c) For an invertible matrix A, show that $\det(A^{-1}) = 1/\det(A)$.