Simon Fraser University

Math 232

Midterm 2
Instructor : Aaron Bradford

Date: 6 July 2007
Time: 11:30 - 12:20

Last Name (print): ________________________ **First Name:** ________________________

Signature: ______________________________ **SFU Email ID:** ______________________

Instructions:

1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Ensure that you have 5 pages of questions.
3. No calculators, notes or books are allowed.
4. Except for question 1, credit will not be given for answers with no explanation.
5. Answer each question in the space provided. Continue on the back of the previous page if necessary.
6. Good luck!

Question	Mark	Maximum
1		3
2		4
3		3
4		4
5		6
6		4
7		5
Total		**29**

1. (**½ point each**) Mark the following statements as either true or false. No explanation is required.

a. ____ $\det(A+B) = \det A + \det B$

b. ____ If all of the entries in A are integers and $\det A = 1$, then all of the entries in A^{-1} are integers.

c. ____ The correspondence $[\vec{x}]_{\mathcal{B}} \mapsto \vec{x}$ is called the co-ordinate mapping.

d. ____ $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

e. ____ It is possible for the null space of a 10×12 matrix A to have dimension 1.

f. ____ The columns of $P_{\mathcal{C} \leftarrow \mathcal{B}}$ are linearly independent.

2. (**3 points – 1 point**) Suppose that $A = \begin{pmatrix} 3 & 6 & 7 \\ 0 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

a. Compute the $(1,2)$-, $(2,3)$- and $(3,1)$-co-factors of A.

b. The remaining six co-factors of A are $C_{11} = 2$, $C_{13} = -4$, $C_{21} = -3$, $C_{22} = -2$, $C_{32} = -6$ and $C_{33} = 6$. Given that $\det A = 2$, find A^{-1}.

3. (**2 points – 1 point**) Consider the matrix $A = \begin{pmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{pmatrix}$.

a. Find det$A$.

b. Is A invertible? Justify.

4. (**3 points – 1 point**) Consider the vector space $\mathbb{P}_1$.

a. Find the change of co-ordinates matrix from the basis $\mathcal{B} = \{1+t, 1-t\}$ to the basis $\mathcal{C} = \{2+t, 1+2t\}$.

b. Given that $[2]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, use your answer from (a) to find $[2]_{\mathcal{C}}$

5. (**1 point – 5 points**)

a. Let V be a vector space. Define what it means for a set $\mathcal{B}$ to be a **basis** for V.

b. Given $A = \begin{pmatrix} 1 & 1 & -3 & 7 \\ 1 & 2 & -4 & 10 \\ 1 & -1 & -1 & 1 \end{pmatrix}$, find a basis for each of Col$A$, Nul$A$, and Row$A$.

6. **(2 points – 2 points)**

a. Define what it means for a set H to be a **subspace** of V.

b. Determine if the set $H = \left\{ \begin{pmatrix} a \\ a^2 \end{pmatrix} : a \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$. Justify.

7. (**1 point – 4 points**) Suppose that V and W are vector spaces, and that $T : V \to W$ is a linear transformation.

a. Define the **kernel** of T.

b. Prove that $\ker T$ is a subspace of V.