

# Math 232, Fall 2007

## Midterm 1

Oct. 5, 2007

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|-------------|--|
| Last Name:  |  |
| First Name: |  |
| SFU ID:     |  |

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. No calculators are allowed.
3. This test is comprised of 6 pages (including cover page)
4. Once the test begins, please check that all pages are intact.
5. Do ALL questions.
6. Clearly explain your answer. No credit will be given for just writing down the answer.
7. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
8. Good luck.

| Question | Points | Score |
|----------|--------|-------|
| 1        | 7      |       |
| 2        | 10     |       |
| 3        | 10     |       |
| 4        | 11     |       |
| 5        | 12     |       |
| Total:   | 50     |       |

1. (7 points) Show that the vectors

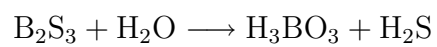
$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

are linearly independent. Show all work.

2. True or False. Justify your answers.

- (a) (2 points) If  $\vec{v}$  and  $\vec{w}$  are two nonzero vectors in  $\mathbb{R}^3$  then they are linearly independent.
- (b) (2 points) If a transformation  $T$  sends  $\vec{0}$  to  $\vec{0}$  then it is a linear transformation.
- (c) (2 points) If a row reduced  $3 \times 4$  matrix has a pivot in every row then its columns span  $\mathbb{R}^3$ .
- (d) (2 points) A linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  is never onto.
- (e) (2 points) A linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  is never one-to-one.

3. (10 points) Balance the chemical equation



using the vector equation approach. Show all work.

4. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be the linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 2x_2 - x_3 \\ 3x_2 + x_1 + 2x_3 \\ x_3 + x_1 \\ x_2 + x_3 \end{bmatrix}$$

- (a) (3 points) Find the matrix of  $T$ . Show all work.
- (b) (4 points) Is  $T$  one-to-one? Justify your answer.
- (c) (4 points) is  $T$  onto? Justify your answer.

5. Consider the system of linear equations:

$$\begin{array}{rcl} x_1 + 2x_2 - x_3 & = & -1 \\ 2x_1 + 5x_2 + (h-1)x_3 & = & -1 \\ 3x_1 + (6-h)x_2 + (-h-4)x_3 & = & -4 \end{array}$$

- (a) (3 points) For which values of  $h$  does the system have no solutions?
- (b) (3 points) For which values of  $h$  does the system have exactly one solution?
- (c) (3 points) For which values of  $h$  does the equation have exactly two solutions?
- (d) (3 points) For which values of  $h$  does the equation have infinitely many solutions?

Justify your answers and show all work.