Math 232, Spring 2007
First Midterm

February 5, 2007, 11:30 – 12:20

Last Name:	
First Name:	
SFU ID:	

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. No calculators are allowed.
3. This test is comprised of 6 pages (including cover page)
4. Once the test begins, please check that all pages are intact.
5. Do ALL questions.
6. Clearly explain your answer. No credit will be given for just writing down the answer.
7. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
8. All the best.

Question	Points	Score
1	8	
2	9	
3	7	
4	6	
Total:	30	

1. We consider the following system of equations:

$$\left\{\begin{array}{rrrrcl} x_1 & +x_2 & +2x_3 & -4x_4 & = & 1 \\ x_1 & +2x_2 & +x_3 & +x_4 & = & 2 \\ 2x_1 & +4x_2 & +2x_3 & -x_4 & = & 1 \end{array}\right.$$

   (a) (1 point) Write down the augmented matrix corresponding to this system.

Answer

   (b) (3 points) Determine the reduced row echelon form of this matrix. Show your work. (use the back of the previous page if you need more room)

Answer

(c) (2 points) Write down the full solution set to the system in parametric form.

Answer

(d) (2 points) Let $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \mathbf{a}_4 = \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix}$. Can you write $\mathbf{b}$ as a linear combination of $\mathbf{a}_1, \ldots, \mathbf{a}_4$? Motivate your answer and, if it is "yes", give such a linear combination. [HINT: Relate the given vectors to the system given in (a)]

Answer

2. (a) (2 points) Give the definition of *linear independence* for a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ of vectors in $\mathbb{R}^n$. Your answer should start with:
"A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ is called linearly independent if..."

Answer

(b) (3 points) Can a set of r vectors in $\mathbb{R}^n$ ever be linearly independent if $n < r$? Prove your statement.

Answer

(c) (4 points) Determine if the set $\left\{ \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix} \right\}$ is linearly independent. Show that your answer is correct.

Answer

3. (a) (4 points) Determine the inverse of the matrix. Show your work.

$$M = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Answer

(b) (3 points) Prove that, for an invertible $n \times n$ matrix A, the linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(\mathbf{x}) = A\mathbf{x}$, is onto.

Answer

4. Mark true or false and give a reason.

   (a) (2 points) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ can never be one-to-one if $n > m$.

Answer

   (b) (2 points) The following map is linear:

$$\begin{array}{ccc} \mathbb{R}^2 & \to & \mathbb{R}^2 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \mapsto & \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \end{pmatrix} \end{array}$$

Answer

   (c) (2 points) A matrix transformation is always an linear transformation.

Answer