Math 232, Fall 2007
Final
Dec. 12, 2007

Last Name:	
First Name:	
SFU ID:	

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. No calculators are allowed.
3. This test is comprised of 18 pages (including cover page)
4. Once the test begins, please check that all pages are intact.
5. Do ALL questions.
6. Clearly explain your answer. No credit will be given for just writing down the answer.
7. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
8. Good luck.

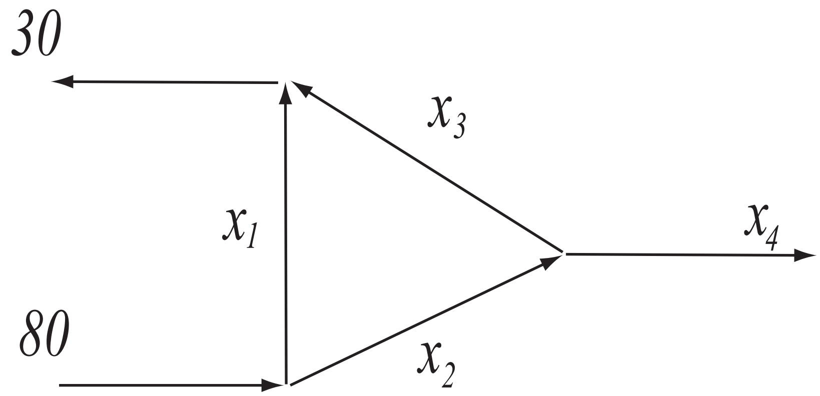
Question	Points	Score
1	8	
2	9	
3	12	
4	9	
5	9	
6	9	
7	11	
8	10	
9	12	
10	11	
Total:	100	

1. Let A be the matrix
$$\begin{pmatrix} 3 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 3 \end{pmatrix}.$$

   (a) (6 points) Compute the characteristic polynomial of A. Show all work. Your final answer should be of the form $ax^3 + bx^2 + cx + d$.

   (b) (2 points) Use part a. to decide whether A is invertible or not.

2. (9 points) Assuming that all the flows are nonnegative in the figure below, what is the largest



possible value for x_3? Show all work.

3. True or False. Justify your answers.

   (a) (2 points) If an $n \times n$ matrix is diagonalizable it has n distinct eigenvalues.

   (b) (2 points) If A is a square matrix and $A\vec{v}$ is in the span of $\vec{v}$, then $\vec{v}$ is an eigenvector of A.

   (c) (2 points) If an $n \times n$ matrix is upper-triangular then its eigenvalues are just the entries along its main diagonal.

   (d) (2 points) If A is a 3×3 matrix and $A^2 = 0$ then $A = 0$.

   (e) (2 points) If a system of equations has at least 2 solutions then it has at least three solutions.

   (f) (2 points) If $\det(A) = 3$ and $\det(B) = 6$ then $\det(A^{-1}B) = 2$.

4. Which of the following sets are subspaces of $\mathbb{R}^4$? Justify your answer.

   (a) (3 points)

$$W_1 = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \ : \ x + 3y = 1 \text{ and } x + 2z = 3 \right\}$$

   (b) (3 points)

$$W_2 = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \ : \ x + 3y = 0 \text{ and } x + 2z = 0 \right\}$$

   (c) (3 points)

$$W_3 = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \ : \ x + 3y = 0 \text{ or } x + 2z = 0 \right\}$$

5. In the table below, data are given for two variables x and y.

x	y
-2	-3
-1	-1
0	1
1	3
2	4

(a) (7 points) Compute the line of best fit for these data. Show all work.

(b) (2 points) What does the line of best fit estimate the value of y will be when $x = 5$.

6. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_2 + 3x_1 \\ x_1 + x_2 \end{bmatrix}$$

and

$$\mathcal{B} = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$$

and

$$\mathcal{C} = \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$$

(a) (7 points) Compute the matrix of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$, $[T]_{\mathcal{B}}^{\mathcal{C}}$. Show all work.

(b) (2 points) if

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

what is $[T(\vec{x})]_{\mathcal{C}}$?

7. (a) (8 points) Find $\mathrm{Col}(A)^{\perp}$ when

$$A = \begin{pmatrix} 1 & 1 & 3 & 5 \\ 1 & 2 & 3 & 6 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & 1 & 4 \end{pmatrix}.$$

Show all work.

(b) (3 points) Use your work in part a. to determine the dimension of $\mathrm{Col}(A)$ and $\mathrm{Nul}(A)$.

8. (a) (8 points) Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 6 \\ -3 \\ 0 \end{bmatrix}$$

onto the Column space of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -1 & -3 \\ 1 & 2 & 3 \end{pmatrix}.$$

Show all work.

(b) (2 points) Find $\vec{y} \in \mathrm{Col}(A)$ and $\vec{z} \in \mathrm{Col}(A)^{\perp}$ such that $\vec{v} = \vec{y} + \vec{z}$.

9. (a) (8 points) Show that $2h+3$ is an eigenvalue of the matrix

$$\begin{pmatrix} 2h+2 & 1 & 0 \\ h+3 & h & 1 \\ 2-h & h-2 & 2 \end{pmatrix}.$$

(b) (4 points) Find an eigenvector corresponding to the eigenvalue $2h+3$. Show all work.

10. Let

$$A = \begin{pmatrix} 11 & -15 \\ 6 & -8 \end{pmatrix}.$$

(a) (6 points) Find an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$. Show all work.

(b) (5 points) Find a formula for A^{100}. (Hint: Use part a. Your formula should be given by a 2×2 matrix whose entries are of the form $C_0 a^{100} + C_1 b^{100}$, where C_0, C_1, a, b are constants.) Show all work.