Math 232, Spring 2007
Final Examination

April 10, 2007, 15:30 – 18:30

Last Name:	
First Name:	
SFU ID:	

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. No calculators are allowed.
3. This test is comprised of 11 pages (including cover page)
4. Once the test begins, please check that all pages are intact.
5. Do ALL questions.
6. Clearly explain your answer. No credit will be given for just writing down the answer.
7. If the answer space provided is not sufficient, write your answer on the back of the previous page. Clearly mark the question number.
8. The last question is a *bonus question*. This question receives no partial credit.
9. All the best.

Question	Points	Score
1	9	
2	9	
3	6	
4	10	
5	8	
6	3	
7	7	
8	8	
Total:	60	

Bonus question

1. Consider the matrix

$$\begin{pmatrix} 2 & 8 & -2 \\ 3 & 13 & -4 \\ -1 & -7 & 3 \end{pmatrix}$$

   (a) (3 points) Compute $\det(A)$.

Answer

   (b) (3 points) Compute A^{-1}.

Answer

   (c) (3 points) Solve the system

$$\left\{ \begin{array}{rrrcr} 2x_1 & +8x_2 & -2x_3 & = & -2 \\ 3x_1 & +13x_2 & -4x_3 & = & -5 \\ -1x_1 & -7x_2 & +3x_3 & = & 5 \end{array} \right.$$

Answer

2. Consider the matrix

$$A = \begin{pmatrix} 10 & -12 \\ 6 & -7 \end{pmatrix}$$

(a) (3 points) Compute the characteristic polynomial of A.

Answer

(b) (4 points) Compute the eigenvalues of A and the corresponding eigenspaces.

Answer

(c) (2 points) Give an invertible matrix P and a diagonal matrix D such that $A = P^{-1}DP$.

Answer

3. Let $W \subset \mathbb{R}^3$ be the subspace spanned by $\{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\}$.

   (a) (3 points) Compute a basis for $W^{\perp}$. Explain your method.

Answer

   (b) (3 points) Compute the orthogonal projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ onto W and onto $W^{\perp}$. Explain your method.

Answer

4. Consider the following basis for $\mathbb{R}^3$: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} \right\}$.

   (a) (2 points) Give the definition of an orthogonal basis.

Answer

   (b) (2 points) Show that the given basis for $\mathbb{R}^3$ is not an orthogonal basis.

Answer

(c) (4 points) Perform the Gram-Schmidt orthogonalisation process on the given basis to obtain an orthogonal basis for $\mathbb{R}^3$. You may rescale vectors if it makes the arithmetic easier for you.

Answer

(d) (2 points) Give the orthogonal projection of $\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$ onto the orthogonal complement of $\text{Span}\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}\}$. Explain your answer.

Answer

5. An experiment has yielded the following measurements for the quantities (x, y): $(0, 1), (1, 3), (2, 3), (3, 1)$. Some theory predicts that $y = \beta_0 + x\beta_1$.

   (a) (2 points) Express the problem of determining β_0, β_1 as a system of linear equations. Is the system consistent? Explain why (not).

Answer

   (b) (2 points) For a system of linear equations $A\mathbf{x} = \mathbf{b}$, describe the *least squares solution* to this system in terms of A, $\mathbf{b}$ and orthogonal projections.

Answer

   (c) (4 points) Compute the least squares line for the points given.

Answer

6. (3 points) Let A be an $n \times m$ matrix. Prove that if $\mathbf{v} \in \text{Nul}(A)$ then $\mathbf{v}$ is orthogonal to all vectors in the row space of A.

Answer

7. We consider the vector space $\mathbb{R}^{3\times 3}$ of 3×3 matrices. We consider the subset of symmetric matrices:

$$V = \{A \in \mathbb{R}^{3\times 3} : A^T = A\}$$

(a) (2 points) What properties should V satisfy to be a subspace?

Answer

(b) (3 points) Prove that V is a subspace.

Answer

(c) (2 points) What dimension does V have? Give a basis for V.

Answer

8. Let $\mathbf{v} \in \mathbb{R}^n$ and consider the map $T : \mathbb{R}^n \to \mathbb{R}$ defined by $T(\mathbf{w}) = \mathbf{w} \cdot \mathbf{v}$.

   (a) (2 points) What properties should T satisfy to be a linear transformation?

Answer

   (b) (3 points) Prove that T is a linear transformation.

Answer

   (c) (3 points) Give the standard matrix of T and describe how you compute it.

Answer

Bonus question: Let $W \subset \mathbb{R}^n$ be a subspace and let $\mathrm{proj}_W : \mathbb{R}^n \to \mathbb{R}^n$ be the orthogonal projection onto W. Prove that proj_W can only have the eigenvalues 0 and 1 and describe the corresponding eigenspaces in terms of W.

Answer