Simon Fraser University
Department of Mathematics
Burnaby Campus

MACM 201-3, Fall 2005
Midterm 2
November 9^{th}, 2005, 12:30 pm – 1:20 pm

Last Name (please print): ______________________

First Name (please print): ______________________

Student Number: ______________________

Signature: ______________________

Instructions:

1. Do NOT open this booklet yet.
2. Fill in the above box NOW.
3. This exam contains 6 pages with a total of 5 questions. Once the exam begins please check your exam is complete.
4. No notes are allowed. Cellphones off!
5. If you run out of space in a problem, use the space on the back of the previous page and indicate where the solution continues.
6. For the first 2 minutes of the test you are NOT ALLOWED to write, except to mark key words of questions. Use this time to read the questions carefully and decide the order in which you will answer them.
7. Full marks will be reserved for answers that are CORRECT IN ALL ESSENTIAL DETAILS, and could be understood by another student without undue effort.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is FORBIDDEN.
9. You MUST stop writing when time is called.

Do not write in this table!	
Question	**Marks**
1	**/8**
2	**/10**
3	**/6**
4	**/8**
5	**/8**
Total	**/40**

1. Let G be an undirected graph or multigraph with no isolated vertices.

   (a) **[2 marks]** Assuming that the definition of a walk is known, what is meant by saying that G has an *Euler circuit*?

   (b) **[2 marks]** State necessary and sufficient conditions for G to have an Euler circuit.

   (c) **[4 marks]** Let s be any vertex of G. Prove in careful detail that, if the conditions you have stated in part (b) are satisfied, there is a circuit of G of nonzero length containing s.

2.

(a) **[5 marks]** Poker chips come in three colours: red, blue and green. Let a_n be the number of ways of stacking n of these chips so that no three consecutive chips are green. Find, with justification, a recurrence relation for a_n. (Do not try to solve the recurrence relation.)

(b) **[5 marks]** The general solution of the recurrence relation $a_{n+2} + b_1 a_{n+1} + b_2 a_n = b_3 2^n$, $n \geq 0$, with b_i constant for $1 \leq i \leq 3$, is $c_1 2^n + c_2(-4)^n + n2^n$. Find b_i for each $1 \leq i \leq 3$.

3. **[6 marks]** Up to isomorphism, how many loop-free undirected graphs are there with four vertices? Justify your answer.

4. Let G be a connected planar graph or multigraph with v vertices, e edges, and r regions determined by a planar embedding of G.

   (a) **[1 mark]** What does it mean for G to be *connected*?

   (b) **[1 mark]** What does it mean for G to be *planar*?

   (c) **[1 mark]** State the relationship between v, e and r given by Euler's Theorem.

   (d) **[5 marks]** Suppose the relationship you have stated in part (c) holds when G contains up to $e - 1$ edges. Let f be an edge joining distinct vertices a and b for which the graph $G - f$ is connected. Prove in careful detail that the relationship holds for G containing e edges.

5. Determine whether the following statements are TRUE or FALSE. In both cases demonstrate conclusively why.

   (a) **[4 marks]** Up to isomorphism, there are exactly three complete bipartite graphs with 6 vertices.

   (b) **[4 marks]** The following graph is planar.