SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Midterm 2

MACM 201 Spring 2008

Instructor: Robert Šámal

March 5, 2008, 12:30 – 13:20

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 5 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	6	
3	14	
4	10	
5	10	
Total	50	

[10] **1.** Solve the following recurrence relation

$$a_{n+1} + 2a_n = 2^n \quad (n \geq 0), \qquad a_0 = 1\,.$$

You may use any of the methods we learned.

[2] **2.** (a) Evaluate $\binom{\frac{2}{3}}{2}$. (Simplify your expression to get a simple fraction.)

[2] (b) Explain why $p(4) = 5$. (That is, list all objects that are counted by $p(4)$.)

[2] (c) For each of the five objects you listed in the previous part, draw the corresponding Ferrers diagram.

3. Find generating functions for the following sequences (include all the necessary computation).

[1] (a) $1, 0, 1, 0, 1, 0, 1, 0, \ldots$

[2] (b) $1, -1, 1, -1, 1, -1, \ldots$

[2] (c) $2, 0, 4, 0, 8, 0, 16, \ldots$

[3] (d) The generating function for the sequence of third powers, $0^3, 1^3, 2^3, 3^3, 4^3, 5^3, \ldots$ is $f(x) = \dfrac{x^3 + 4x^2 + x}{(1-x)^4}$. (You don't need to verify this.) Use this fact to find the generating function for $0, 0, -1^3, 2^3, -4^3, 5^3, \ldots$

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[2] (e) Find $[x^2](\frac{2}{x} + 3x)^{10}$.

[4] (f) Find $[x^{60}]\frac{1+3x+x^2}{(1-x^3)^3}$.

4. We want to distribute 101 cookies to three children: Adam, Beata, and Colin.

- Adam always eats two cookies at the same time, so we want to give him an even number of cookies, but not more than 20.
- Beata has a birthday today, so we want to give her at least 20 cookies.
- Colin can get any number of cookies.

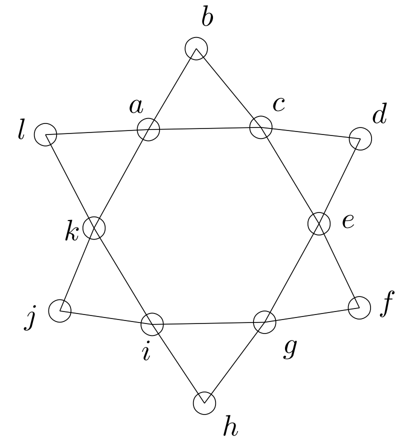
Find out how many ways we have to distribute the cookies.

[3] (a) Express the answer as a coefficient of some power of x in an appropriate generating function $f(x)$. (Don't forget to explain why your formula is correct.)

[4] (b) Express $f(x)$ as $\frac{\text{a polynomial}}{(1-x^2)^3}$.

[3] (c) Use the binomial theorem to get the final answer.

5. Consider the graph on the figure. In the first four questions you don't need to provide any explanation.



[1] (a) How many vertices does the graph have?

[1] (b) How many edges does the graph have?

[1] (c) How many cycles of length 4 does the graph have?

[1] (d) How many cycles of length 3 does the graph have?

In the following parts we will consider various walks in the graph. (You may use any of the several ways to specify a walk.) In parts (f) and (g) explain why the walk you provided is not a path/a trail.

[2] (e) Write down an a-d path.

[2] (f) Write down an a-d trail that is not a path.

[2] (g) Write down an a-d walk that is not a trail.