

SIMON FRASER UNIVERSITY
DEPARTMENT OF MATHEMATICS

Midterm 2

MACM 201 Summer 2007

Instructor: Robert Šámal

July 4, 2007, 12:30 – 13:20

Name: _____ (please print)
family name *given name*

SFU ID: _____
student number *SFU-email*

Signature: _____

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 5 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	12	
3	10	
4	10	
5	10	
Total	50	

[2] 1. (a) Evaluate $\binom{-\frac{3}{4}}{2}$.

[2] (b) Explain why $p(4) = 5$. (That is, list all objects that are counted by $p(4)$.)

[4] (c) Let $a_n = 2$ and $b_n = n$ for all $n \geq 0$. Suppose that the sequence c_0, c_1, c_2, \dots is the convolution of a_0, a_1, a_2, \dots and b_0, b_1, b_2, \dots . What is c_n ?

2. Find generating functions for the following sequences (include all the necessary computation).

[1] (a) $0, 1, 0, 1, 0, 1, 0, 1, \dots$

[2] (b) $1, 0, 2, 0, 4, 0, 8, 0, 16, \dots$

[3] (c) $0, -1, 2, -3, 4, -5, 6, -7, \dots$

[2] (d) Find the coefficient of x^6 in $(1 + 2x)^{10}$.

[4] (e) Find the coefficient of x^{54} in $x^5(x^3 + x^7 + x^{11} + x^{15} + \dots)^3$.

[10] **3.** Let a_n be given by the following recurrence relation

$$a_n - 2a_{n-1} = 3^n \quad (n \geq 1), \quad a_0 = 3.$$

Find the generating function for the sequence a_0, a_1, a_2, \dots .

4. This question concerns finding the number of solutions to the equation

$$a + b + c = 100,$$

where a, b, c are nonnegative integers such that

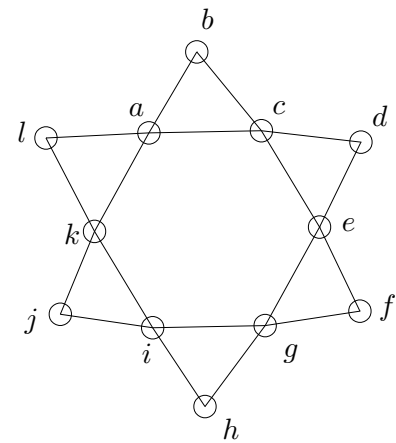
- a is even,
- b is odd, and
- $c \geq 5$.

[3] (a) Express the answer as a coefficient of some power of x in an appropriate generating function $f(x)$.

[4] (b) Express $f(x)$ as $\frac{\text{a polynomial}}{(1-x^2)^3}$.

[3] (c) Use the binomial theorem to get the final answer.

5. Consider various walks in the graph on the figure. In parts (b), (c), (e) explain why the walk you provided is not a path, a trail, and a cycle, respectively. (You may use any of the several ways to specify a walk.)



- [2] (a) Write down an a - f path.
- [2] (b) Write down an a - f trail that is not a path.
- [2] (c) Write down an a - f walk that is not a trail.
- [2] (d) Write down an a - a cycle.
- [2] (e) Write down an a - a circuit that is not a cycle.