SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

# Midterm 2

**MACM 201** Spring 2007

Instructor: Robert Šámal

March 7, 2007, 12:30 – 13:20

**Name:** ______________________________ (please print)

*family name* *given name*

**SFU ID:** ______________________________

*student number* *SFU-email*

**Signature:** ______________________________

**Instructions:**

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 5 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

| Question | Maximum | Score |
|---|---|---|
| 1 | 2 | |
| 2 | 7 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 39 | |

[2] **1.** Solve the following recurrence relation

$$a_n = 10a_{n-1} \quad (n \geq 1), \qquad a_2 = 5\,.$$

[5] **2.** (a) Solve the following recurrence relation. (More space is available on the next page.)

$$a_{n+2} + 4a_{n+1} + 4a_n = 0 \quad (n \geq 0), \qquad a_0 = 1, a_1 = 4\,.$$

[2] (b) Check the answer for $n = 2$.

[10] **3.** For $n \geq 0$ let $a_n$ be the number of ways $n$ can be expressed as a sum of $\mathbf{odd}$ integers, if order matters. For example, $a_4 = 3$, as $4 = 3 + 1 = 1 + 3 = 1 + 1 + 1 + 1$.
Find a recurrence relation for $a_n$ (do not solve it).

**4.** Find $\sum_{k=1}^{n} k2^k$ be means of recurrence relations. (Use the back of the previous page if more space is needed.)

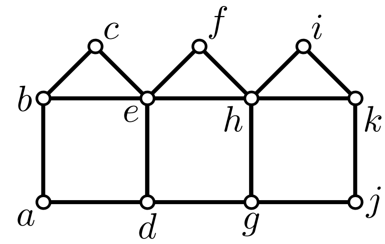
[2] (a) Put $a_n = \sum_{k=1}^{n} k2^k$ and write a recurrence relation for $a_n$.

[2] (b) Find $a_1$, $a_2$, and $a_3$ 'by hand'.

[5] (c) Solve the recurrence relation found in (a).

[1] (d) Verify the solution for $n = 1, 2, 3$.

**5.** Consider various walks in the graph on the figure. In parts (b), (c), (e) explain why the walk you provided is not a path, a trail, and a cycle, respectively.




[2] (a) Write down an $a$-$f$ path.

[2] (b) Write down an $a$-$f$ trail that is not a path.

[2] (c) Write down an $a$-$f$ walk that is not a trail.

[2] (d) Write down an $a$-$a$ cycle.

[2] (e) Write down an $a$-$a$ circuit (closed trail) that is not a cycle.