SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Midterm 1

MACM 201 Spring 2008

Instructor: Robert Šámal

February 6, 2008, 12:30 – 13:20

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 6 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. Let S be a (finite) set, and c_1, c_2, c_3 conditions on the elements of S.

[1] (a) State the Principle of Inclusion and Exclusion, and explain the used notation. Don't use '...', nor '$\sum$'.

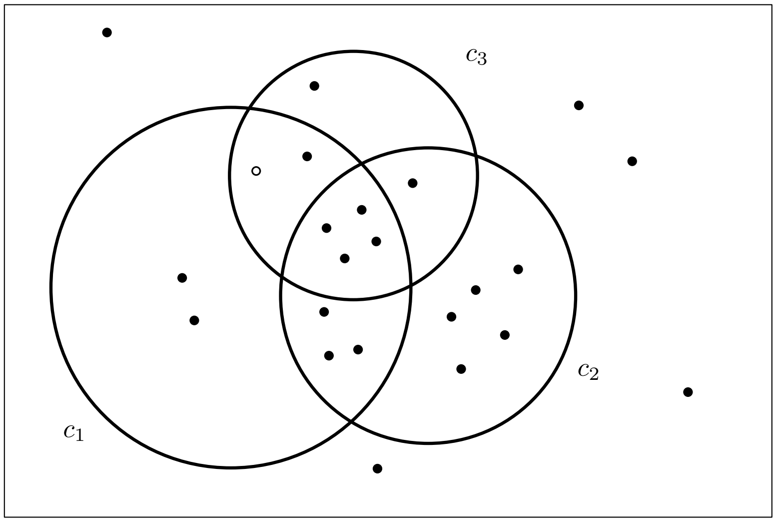
[2] (b) Decide whether the following formulas are true or false, and if false, correct them!

1. $N(\, c_2\,) = N - N(\,\overline{c_2}\,)$

2. $N(\,\overline{c_2}\; c_3\,) = N(\, c_1\,) - N(\, c_2\; c_3\,)$

[2] (c) Which elements of S are counted by $N - N(\,\overline{c_1}\;\overline{c_2}\;\overline{c_3}\,) - N(\, c_1\;\overline{c_2}\;\overline{c_3}\,)$?

In the next two parts of this question, the set S and the conditions c_1, c_2, c_3 are as on the figure (both full and empty points count).



[4] (d) Fill in the following table:

$N(c_1)$	$N(c_2)$	$N(c_3)$	$N(\overline{c_1}\ c_2)$	$N(c_2\ c_3)$	S_1	E_2	L_2

[1] (e) How many times is the element of S denoted by the empty ("white") dot in the figure counted in the expression S_1?

2. In how many ways can five A's, five B's and five C's be arranged to make a 15-letter word, if we require that there is

[7] (a) no consecutive fivetuple of the same letter?

[3] (b) exactly one consecutive fivetuple of the same letter?

3. How many solutions there are to the equation

$$x_1 + x_2 + x_3 + x_4 = 10$$

such that $x_1, \ldots, x_4$ are integers and

[3] (a) $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$?

[7] (b) $0 \leq x_1 \leq 4$, $0 \leq x_2 \leq 4$, $0 \leq x_3 \leq 4$, $0 \leq x_4 \leq 4$?

4. We need to arrange cars on a parking lot with n parking places, which form a single row. There are big cars (occupying two adjacent places) and small ones (occupying just one). Big cars come in two colors, black and silver. Small cars come in three colors, red, green, and blue. There is an unlimited supply of each of the five types. Let a_n be the number of ways to arrange cars on the n parking spaces, where

- no empty spaces should be left, and
- we only care about the "color pattern"; that is, all cars of the same color are considered identical.

[1] (a) Determine a_1.

[1] (b) Determine a_2.

[3] (c) How many of the considered car arrangements have a blue car at the last (n-th) parking space? Express your answer using the numbers $a_1, \ldots, a_{n-1}$ (not necessarily all of them).

[5] (d) Determine a recurrence relation for a_n. (You don't need to solve it.)

[8] **5.** (a) Solve the following recurrence relation.

$$a_{n+2} + 2a_{n+1} - 8a_n = 0 \quad (n \geq 0), \qquad a_0 = 0, a_1 = 1\,.$$

[2] (b) Check the answer for $n = 2$.