SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Midterm 1

MACM 201 Summer 2007

Instructor: Robert Šámal

June 6, 2007, 12:30 – 13:20

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

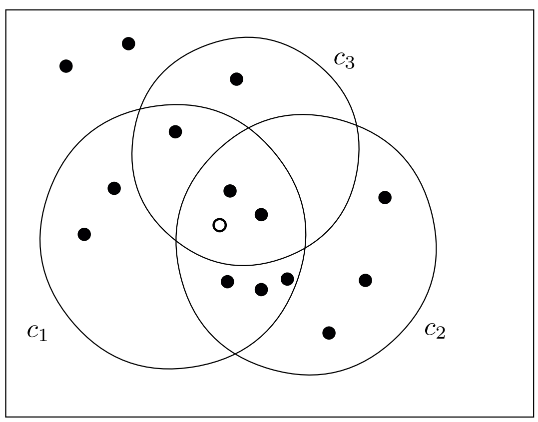
1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 7 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. Let S be a (finite) set, $c_1, \ldots, c_t$ conditions on the elements of S.

[1] (a) Suppose $t = 3$. State the Principle of Inclusion and Exclusion, and explain the used notation. Don't use '$\ldots$', nor '$\sum$'.

[3] (b) Let $t = 3$ again, and let S be as on the figure (both full and empty points count). Fill in the following table:



$N(c_1)$	$N(c_2)$	$N(c_3)$	$N(c_1c_2)$	$N(\overline{c_2}\ c_3)$	S_1

[1] (c) With the same set S as in the previous part, how many times is the element of S denoted by empty circle counted in the sum $S_2 = \sum_{1 \le i < j \le t} N(c_ic_j)$?

Decide if the following formulas are true or false, and if false, correct them!

[1] (d) $N(\,\overline{c_1}\,) = N - N(\,c_1\,)$

[1] (e) $N(\,\overline{c_1}\ c_2\,) = N(\,c_2\,) - N(\,c_1\ c_2\,)$

[1] (f) $N(\,c_1\ c_2\,) = N(\,c_2\,) - N(\,c_1\ \overline{c_2}\,)$

[2] (g) $N(\,c_1\ c_2\,) = N - N(\,\overline{c_1}\ \overline{c_2}\ \overline{c_3}\,) - N(\,\overline{c_1}\ \overline{c_2}\ c_3\,)$

2. Use Principle of Inclusion and Exclusion to determine, how many integers n are there so that $1 \leq n \leq 211$ and

[5] (a) n is not divisible by any of 2, 5, 7?

[5] (b) n is divisible by at least two of 2, 5, 7?

[10] **3.** Two giants, three witches, and seven dwarfs are going to take a group photograph, that is they need to arrange in a single row. To make the photograph look interesting, they want that

- the two giants don't stand next to each other,
- the three witches don't stand all together, **and**
- the seven dwarfs don't stand all together.

For example, the following is a proper arrangement: $G_2D_7D_2D_3W_3D_5D_6D_4D_1W_2W_1G_1$.
How many different photographs can they make in this way?

[3] **4.** (a) Solve the following recurrence relation

$$a_n = 4a_{n-1} \quad (n \geq 1), \qquad a_2 = 8\,.$$

[5] (b) Solve the following recurrence relation. (More space is available on the next page.)

$$a_{n+2} - 6a_{n+1} + 8a_n = 0 \quad (n \geq 0), \qquad a_0 = 1, a_1 = 3\,.$$

[2] (c) Check the answer for $n = 2$.

5. Find $\sum_{k=1}^{n} \frac{k}{2^k}$ be means of recurrence relations. (Use the back of the previous page if more space is needed.)

[2] (a) Put $a_n = \sum_{k=1}^{n} \frac{k}{2^k}$ and write a recurrence relation for a_n.

[2] (b) Find a_1, a_2, and a_3 directly.

[5] (c) Solve the recurrence relation found in (a).

[1] (d) Verify the solution for $n = 1, 2, 3$.