SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Midterm 1

MACM 201 Spring 2007

Instructor: Robert Šámal

February 7, 2007, 12:30 – 13:20

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

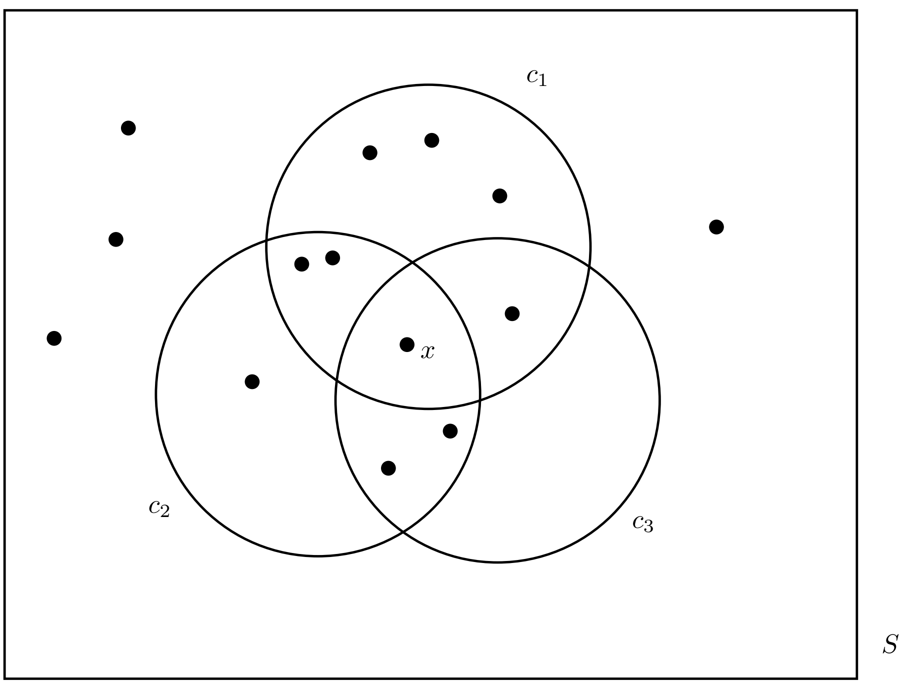
1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 5 questions on 7 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. The only exception is your formula sheet—a one-sided sheet of paper.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	16	
4	14	
5	10	
Total	60	

1. Let S be a (finite) set, $c_1, \ldots, c_t$ conditions on the elements of S.

[1] (a) Suppose $t = 4$. State the Principle of Inclusion and Exclusion, and explain the used notation. Don't use '$\ldots$', nor '$\sum$'.

[3] (b) Let $t = 3$ and let S be as on the figure, fill in the following table:



$N(c_1)$	$N(c_2)$	$N(c_3)$	$N(c_1c_2)$	$N(c_1\overline{c_2})$	S_2

[1] (c) With the same set S as in the previous part, how many times is the element x counted in the sum $S_2 = \sum_{1 \le i < j \le t} N(c_ic_j)$?

[5] (d) Suppose $t = 6$. Write and justify a formula for $N(c_1\ \overline{c_2}\ \overline{c_3}\ c_4 c_5\ \overline{c_6})$! In the formula you may use only the numbers that appear in the Principle of Inclusion and Exclusion, e.g., $N(c_1)$, $N(c_1 c_2)$, but not $N(c_1 \overline{c_2})$.

2. Use Principle of Inclusion and Exclusion to determine, how many integers n are there (you should write down an expression but don't have to evaluate it) so that $1 \leq n \leq 2007$ and

[5] (a) n is not a second, third, or fifth power?

[5] (b) n is not a second, third, or fourth power?

3. How many ways are there to distribute n undistinguishable objects among k people if

[2] (a) everybody can receive any amount of them?

[4] (b) everybody should receive at least one of them?

[10] (c) everybody shoud receive at most $n/3$ of them? (Suppose n is divisible by 3.)

4. Find generating functions for the following sequences (include all the necessary computation).

[1] (a) $0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, \ldots$

[2] (b) $0, 0, 2, 0, -2, 0, 2, 0, -2, 0, 2, 0, -2, \ldots$

[3] (c) $4, 5, 6, 7, 8, 0, 16, 0, 32, 0, 64, 0, 128, 0, 256, 0, 512, 0, \ldots$

[3] (d) $0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, \ldots$

[1] (e) Evaluate $\binom{1/3}{3}$.

[4] (f) Find the coefficient of x^{26} in the following function: $x^3(x^3 + x^5 + x^7 + x^9 + \cdots)^5$.

[10] **5.** A group of 21 people needs to split into four groups during a trip, one group will continue by foot, another by tandem bikes, the third by canoe, and the fourth by kayak. They need to respect the following

- the number of people that go by canoes is even and at least 2;
- the number of people that go by kayaks is even;
- the number of people that go by tandem bikes is even and at least 4.
- the number of people that go by foot is at least 2 and at most 6

Suppose we only care about how many canoes, kayaks and bikes will be used (each canoe, kayak and tandem bike will cary exactly two people). How many different possibilities are there? Use generating functions.