Simon Fraser University

MACM 201
Summer 2000
Final Examination

Instructor: V. Jungic Date: August 3, 2000

Student: _____

Student number: _____

Signature: _____

Instructions

1. Fill in the information above.
2. Please do not open the examination booklet until you are told to do so.
3. Do all your work in this test booklet. Show all your work.
4. Please no books, no notes, and no programmable calculators.

1	2	3	4	5	6	7	8	9	10	11	12	Total	Grade

1. (a) **[5 marks]** Let $f(x) = 3x^3 + 2x + 1$ and $g(x) = x^3$. Show that

$$f(x) \text{ is } O(g(x))$$

and

$$g(x) \text{ is } O(f(x)),$$

i.e., show that

$$f(x) \text{ is } \Theta(g(x)).$$

(b) **[5]** True or false:

$$x^2 \text{ is } O(x^3)?$$

Justify your answer.

2. (a) **[6]** Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one or two stairs at a time?

   (b) **[2]** What are the initial conditions?

   (c) **[6]** Solve the recurrence relation you found in (a) with the initial conditions from (b).

   (d) **[2]** In how many ways can this person climb a flight of twenty stairs?

3. [**7**] Find a closed form for the generating function for each of the following sequences.

   (a) $-1, -1, -1, -1, -1, -1, -1, 0, 0, 0, \ldots$

   (b) $1, 3, 9, 27, 81, 243, 729, \ldots$

   (c) $-3, 3, -3, 3, -3, 3, \ldots$

4. [**12**] Consider the following relations on the set of positive integers

$$\begin{aligned} R_1 &= \{(x,y)|\ x+y>10\} \\ R_2 &= \{(x,y)|\ x \text{ divides } y\} \\ R_3 &= \{(x,y)|\ \gcd(x,y)=1\} \\ R_4 &= \{(x,y)|\ x \text{ and } y \text{ have the same prime divisors}\} \end{aligned}$$

(a) Which of these relations are reflexive?

(b) Which of these relations are symmetric?

(c) Which of these relations are antisymmetric ?

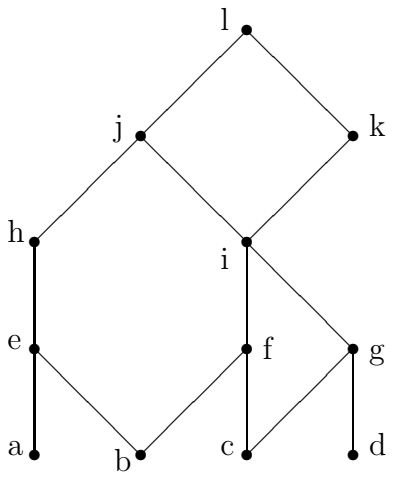
(d) Which of these relations are transitive?

Justify your answers.

5. (a) **[5]** Give a formula how to count onto functions from a set with m elements to a set with n elements.

   (b) **[5]** In how many ways can eight distinct balls be distributed into three distinct urns if each urn must contain at least one ball?

6. [**10**] Consider the poset with the following Hasse diagram.



(a) Find all maximal elements of the poset..

(b) Find all minimal elements of the poset

(c) Find the least element of the poset if it exists, or show that it does not exist.

(d) Find the greatest element of the poset if it exists, or show that it does not exist.

(e) What is the greatest lower bound of the set $\{a, b, c\}$?

(f) What is the least upper bound of the set $\{a, b, c\}$?

7. (a) **[12]** Draw the following graphs. If a graph is planar draw its planar representation. For each graph find its chromatic number. Justify your answer by an appropriate coloring.

   i. K_5
   ii. C_6
   iii. W_5
   iv. Q_4
   v. $K_{4,4}$
   vi. Petersen graph.

(b) **[5]** Define an Euler path and a Hamilton circuit.
Fill out the following table.

Graph	Euler Circuit (Yes/No)	Euler Path (Yes/No)	Hamilton Circuit (Yes/No)	Hamilton Path (Yes/No)
K_5				
C_6				
W_5				
Q_4				
$K_{4,4}$				
$K_{5,4}$				
$K_{5,2}$				

8. (a) **[5]** Draw all non-isomorphic trees with 6 vertices.

   (b) **[5]** Find a planar graph with 8 vertices in which every vertex has degree 3. Draw a planar representation of it.

   Find a non-planar graph with 8 vertices and every vertex has degree 3. Prove that it is not planar.

9. [**10**] Prove or disapprove the following for a weighted graph $G = (V, E)$, where $V = \{v_0, v_1, \ldots, v_n\}$ and $e_1 \in E$ with

$$(\forall e \in E - \{e_1\}) \;\; \text{wt}(e_1) < \;\; \text{wt}(e).$$

If Dijkstra's algorithm is applied to G and the shortest distance $d(v_0, v_i)$ is computed for each vertex v_i, $1 \leq i \leq n$, then there exists a vertex v_j, for some $1 \leq j \leq n$, where the edge e_1 is used in the shortest path from v_0 to v_j.

10. (a) **[2]** Define a tree.

    (b) **[10]** Prove that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

11. (a) **[2]** Represent the expression

$$((x-2) \uparrow 4)/(y-3*x)+7$$

using a binary tree.

(b) **[4]** Write the expression from (a) in

i. prefix notation,

ii. postfix notation.

(c) **[4]** What is the value of the following

i. prefix expression

$$* \; + \; 3 \; + \; 3 \; \uparrow \; 3 \; + \; 3 \; 3 \; 3$$

ii. postfix expression

$$3 \; 2 \; * \; 2 \; \uparrow \; 5 \; 3 \; - \; 8 \; 4 \; / \; * \; -.$$

12. (a) **[2]** Define a simple graph.

    (b) **[2]** Define a connected graph.

    (c) **[2]** Define a spanning tree.

    (d) **[10]** Prove that a simple graph is connected if and only if it has a spanning tree.