SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Final Exam

MACM 201 Spring 2008

Instructor: Robert Šámal

April 14, 2008, 12:00 – 3:00

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable. (Don't evaluate simple numerical expressions involving large numbers, $2^{10} + 3^9$ is as good answer as 20,707.)
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has $10 + 1$ questions on 14 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Bonus	10	
Total	100	

1. A group of 25 students is deciding which courses each of them will take. Suppose that there are only 4 courses available, each student may enroll in any number of them, including 0. (Note: courses are distinguishable, students are distinguishable!)

[3] (a) How many possibilities are there?

[7] (b) How many of the possibilities have at least one student enrolled in each of the 4 courses?

[10] **2.** Ten families (each with five members—mother, father and three children) are going to visit a cinema. In how many ways can they form a queue if no family is standing "together" (that is its members don't occupy five consecutive places in the queue)?

3. Ten families (each with five members) are going to have a dinner (after seeing the movie in question 2 :–)). Each family will be occupying one table for five (there is exactly ten of them available). Each such table has five chairs, each of a different color (red, green, blue, white, and yellow). So everybody's position is described by the number of the table and the color of the chair.
You don't need to provide numerical answers, an (exact) formula involving factorials is just fine.

[2] (a) How many different seating arrangements are there?

[4] (b) Suppose that the people decided to change the seating before they eat the dessert (to enjoy a view from another window). That is, each family moves to a different table, and everybody chooses some chair. How many possibilities are there for this new seating arrangement?

[4] (c) The same question as in the previous part, but now everybody wants to sit on a different-colored chair after the change. How many possibilities are there this time?

[10] **4.** Solve the following recurrence relation

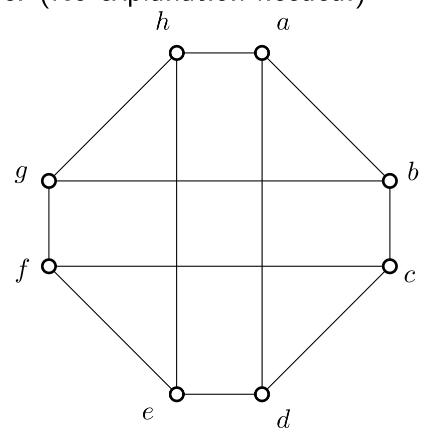
$$a_{n+2} - 4a_{n+1} + 3a_n = n \quad (n \geq 0), \qquad a_0 = 1, a_1 = 1\,.$$

[10] **5.** Find the number of solutions to

$$a + 3b + 6c = 211\,,$$

so that a, b, c are integers, $a \geq 3$, $b \geq 2$, and $c \geq 1$. Use generating functions.

6. Answer the following questions for the graph in the figure. (No explanation needed.)



[1] (a) How many vertices does the graph have?

[1] (b) How many edges does the graph have?

[1] (c) If the graph was drawn without crossings, how many faces would it have?

[1] (d) Does the graph contain a Hamilton cycle?

[1] (e) Does the graph contain an Euler trail?

[2] (f) How many subgraphs contain all outside edges, that is, all of the edges $ab, bc, cd, de, ef, fg, gh$, and ha.

[2] (g) What is the minimum number of edges in a connected spanning subgraph?

[1] (h) What is the distance (length of a shortest path) between a and g?

7. Decide, what is the maximal length of a

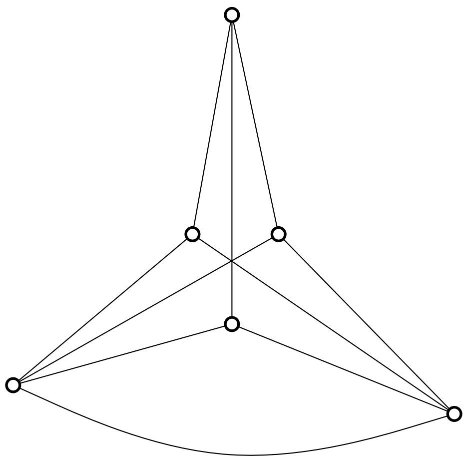
1. cycle
2. circuit

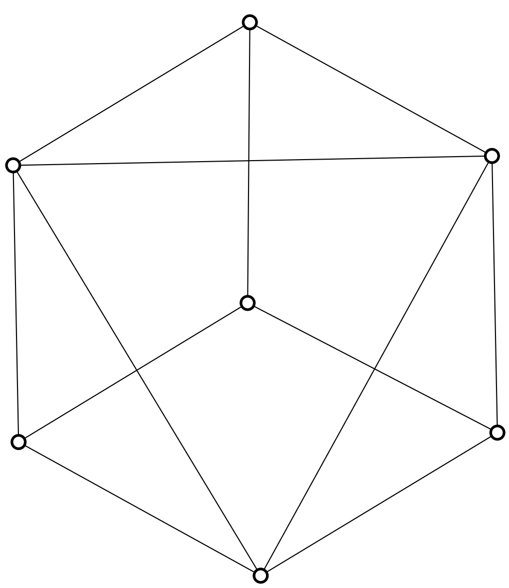
in the following graphs. Explain your results!

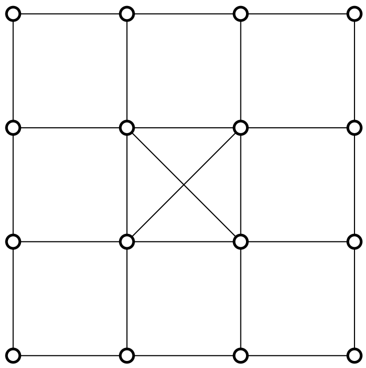
[5] (a) $K_{4,6}$

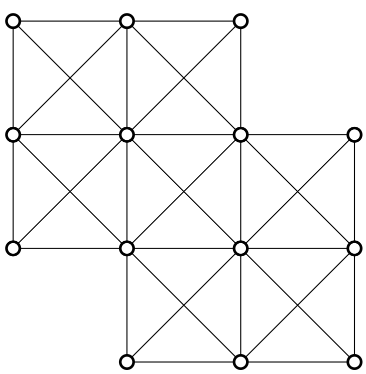
[5] (b) $K_{5,5}$

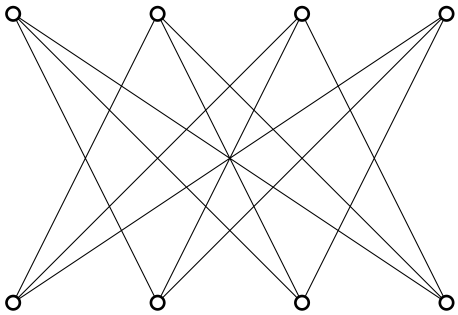
[10] **8.** Are the following graphs planar? If yes, draw the graph without crossings, if not, explain why not. [2 marks for each graph]



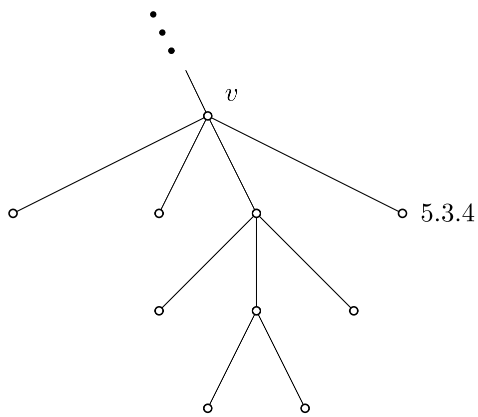








[4] **9.** (a) In the figure is *a part of* a rooted tree. (However, for each vertex in the figure all of its children are shown.) You are given the label of one of the vertices according to the universal address system. Determine the labels of all the other vertices in the figure. What is the level of vertex v?



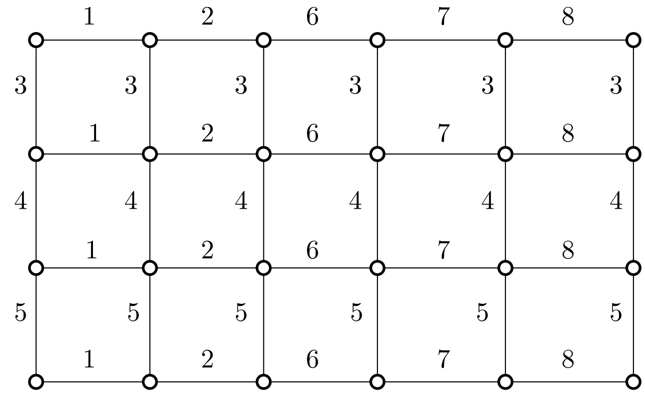
[2] (b) Construct the binary rooted tree for the following arithmetic expression:

$$(1 * 6 + 3) * (2 + 3 * 4)$$

[2] (c) Use the postorder traversal to write the expression in postfix notation.

[2] (d) Use the preorder traversal to write the expression in prefix notation.

10. We will be considering spanning trees in the following graph, with weights as given in the figure.



[1] (a) How many edges can a spanning tree of this graph have?

[9] (b) Apply Kruskal's algorithm to find a minimal spanning tree.

- Emphasize edges of the tree, and label them e_1, e_2, $\ldots$, e_k in the order you add them to the tree (k is the number you found in part (a)).
- Write the weight of the tree found.
- Explain in detail how did you choose e_1 and e_k.

[10] **Bonus.** Decide whether the graph on the figure contains a Hamilton cycle.
Warning 1: It is not easy.
Warning 2: Partial credit will be assigned only for substantial part of the solution.
If you have other questions to work upon, they are likely to bring you more points.