SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Final Exam

MACM 201 Summer 2007

Instructor: Robert Šámal

August 7, 2007, 8:30 – 11:30

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable. (Don't evaluate simple numerical expressions involving large numbers, $2^{10} + 3^9$ is as good answer as 20,707.)
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has $10 + 1$ questions on 14 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Bonus	10	
Total	100	

1. Suppose we want to distribute 10 (distinct) books among 4 students.

[5] (a) How many ways are there to do this if we want everyone to get at least one book?

[5] (b) How many ways are there to do this if we want exactly two people to get nothing? At least two people to get nothing?

[10] **2.** A deck of cards contains 52 cards—4 of each of the 13 ranks. What is the number of ways to order the cards in a line, so that no 4 cards of the same rank appear as a consecutive group? (For example, we don't want 10 J 9 9 9 9 K A . . .)

[10] **3.** Solve the following recurrence relation

$$a_{n+2} = a_{n+1} + 2a_n + 2n + 1 \quad (n \geq 0), \qquad a_0 = 0, a_1 = 1\,.$$

4. Find generating functions for the following sequences (include all the necessary computation).

[1] (a) $1, 2, 4, 8, 16, \ldots$

[1] (b) $1+1, 1+a, 1+a^2, 1+a^3, 1+a^4, \ldots$ (a is a parameter)

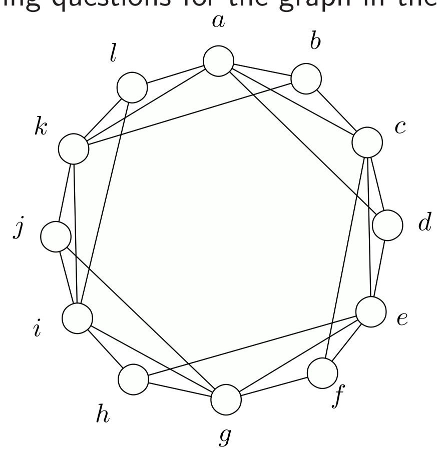
[2] (c) $0, 1, 2, 3, 4, 5, 6, 7, \ldots$

[3] (d) Find the coefficient of x^{14} in $(x^2 + \frac{2}{x})^{40}$.

[3] (e) Let a_n be the number of ways to write n as a sum of distinct odd positive integers (disregarding the order). What is the generating function for the sequence $a_0, a_1, a_2, \ldots$? (Explain what various parts of the function correspond to.)

[10] **5.** We need to distribute 120 envelopes among 4 students so that everybody gets at least 6 and at most 40 envelopes. How many ways are there to do this?

6. Answer the following questions for the graph in the figure. (No explanation needed.)



[1] (a) How many vertices does the graph have?

[1] (b) How many edges does the graph have?

[1] (c) If the graph was drawn without crossings, how many faces would it have?

[2] (d) How many *induced* subgraphs contain the edge ab?

[2] (e) How many *spanning* subgraphs contain the edge ab?

[1] (f) Specify a subgraph that is not induced.

[1] (g) How many edges does a spanning tree have?

[1] (h) What is the distance (length of a shortest path) between a and g?

7. Decide, what is the maximal length of a

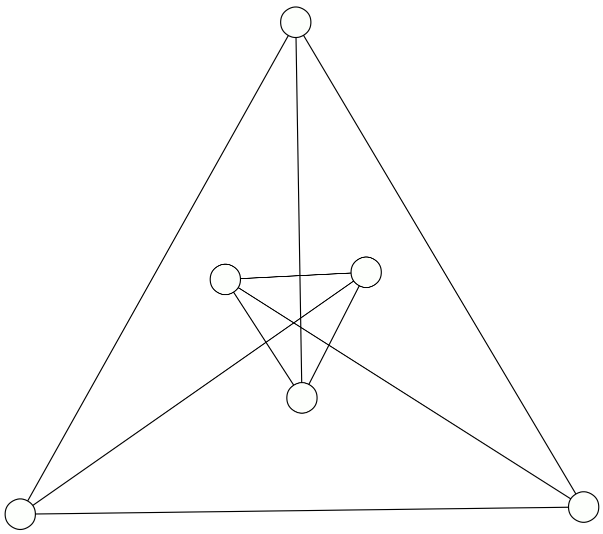
1. cycle
2. circuit

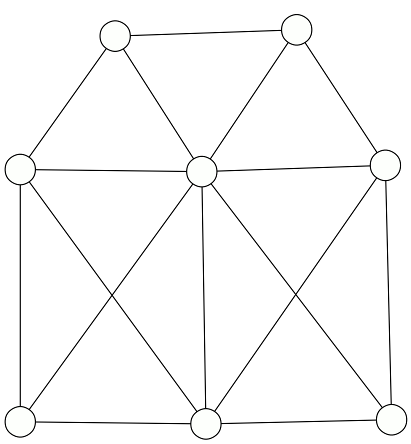
in the following graphs. Explain your results!

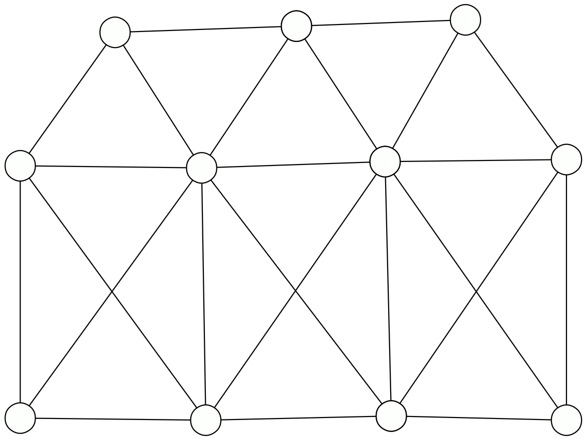
[5] (a) K_9

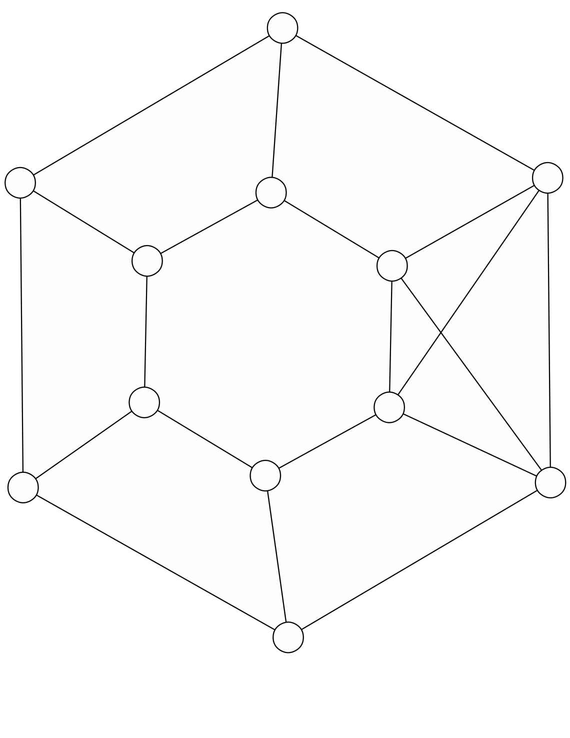
[5] (b) K_{10}

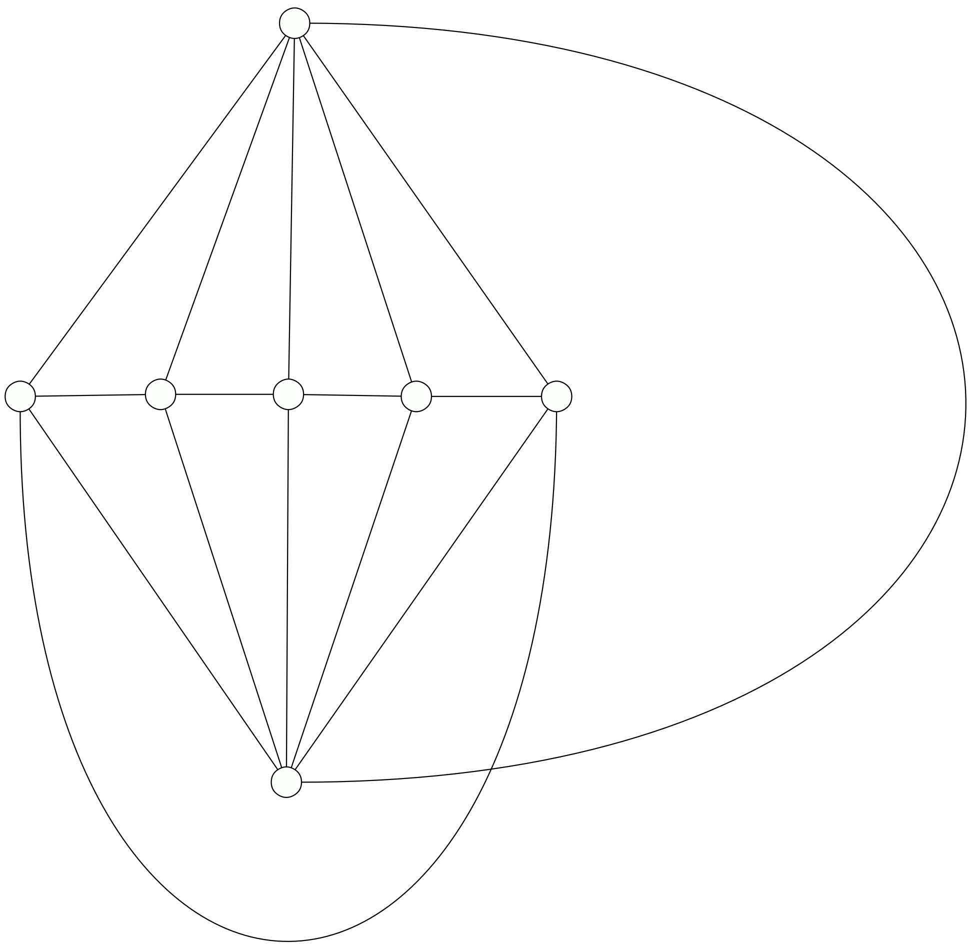
[10] **8.** Are the following graphs planar? If yes, draw the graph without crossings, if not, explain why not. [2 marks for each graph]



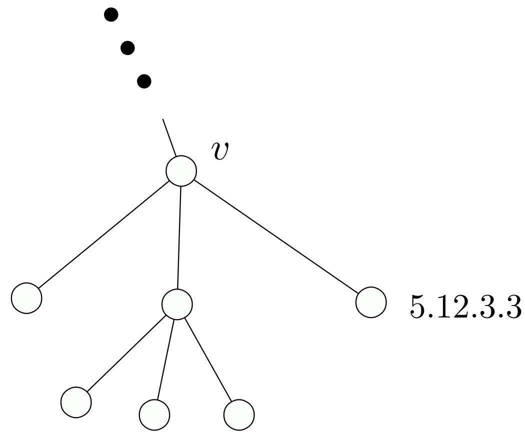








[4] **9.** (a) In the figure is *a part of* a rooted tree. (However, for each vertex in the figure all of its children are shown.) You are given the label of one of the vertices according to the universal address system. Determine the labels of the other vertices, and the labels of the ancestors of the vertex v (these ancestors are not shown on the figure).



[2] (b) Construct the binary rooted tree for the following arithmetic expression:

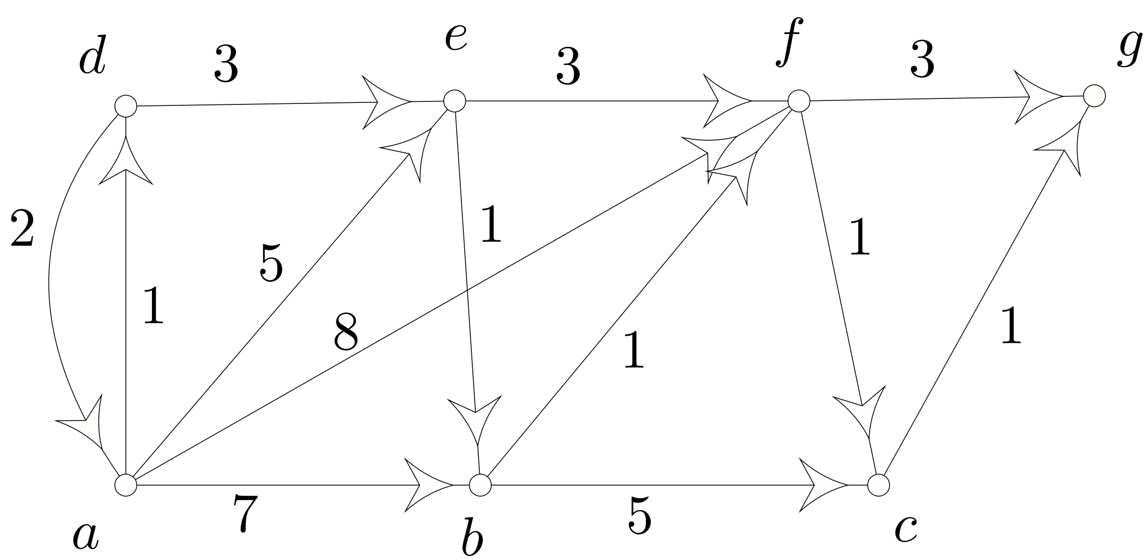
$$(3+5)*((2*3)\,\hat{}\,2+1)$$

[2] (c) Use the postorder traversal to write the expression in postfix notation.

[2] (d) Use the preorder traversal to write the expression in prefix notation.

[10] **10.** Apply Dijkstra's algorithm to the following undirected graph in order to find the shortest path from a to each of the other vertices.

- Fill each of the determined distances in the table below and provide a shortest path from a to g.
- Label the vertices v_0, ..., v_6 according to the algorithm (in the order in which you reached the "final" decision about the distance of the vertex from a).
- Explain in detail the first three steps of the algorithm (that is, all considerations before v_3 was determined).



a	b	c	d	e	f	g

Bonus. Let $m, n \geq 1$ be integers. Let $G_{m,n}$ be the following graph. The vertex-set is $V = \{(x, y) \mid 1 \leq x \leq m, 1 \leq y \leq n\}$. Two vertices (x_1, y_1) and (x_2, y_2) are adjacent if

- $y_1 = y_2$ or
- $x_1 = x_2$ and $|y_1 - y_2| \in \{1, n-1\}$

[5] (a) For which integers m, n does $G_{m,n}$ have an Euler circuit? Explain!

[5] (b) For which integers m, n does $G_{m,n}$ have a Hamilton cycle? Explain!