SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS

Final Exam

MACM 201 Spring 2007

Instructor: Robert Šámal

April 11, 2007, 8:30 – 11:30

Name: ______________________________ (please print)

family name *given name*

SFU ID: ______________________________

student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable. (Don't evaluate simple numerical expressions involving large numbers, $2^{10} + 3^9$ is as good answer as 20,707.)
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 10 questions on 13 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

[10] **1.** Suppose everyone can choose one of the 12 calendar months to take their holidays. If each person in a group of 30 does this, how many ways will there be so that there is at least 1 month when none of them will be taking holidays?

2. We say a word W contains a word W' if W' is included in W without gaps: e.g., the word KAYAK contains YAK but not KYK. Consider all words obtained by reordering the letters of the word ADVENTURE. How many of these words contain

[5] (a) none of the words VAN, AD, TEN, RED?

[5] (b) at least two of those words?

3. Fifty different books (10 on each of five topics) are placed on five bookshelves (each topic on one of the shelves). All books are removed from the shelves and then put back, again all books on the same topic are placed on one shelf. How many ways are there to choose the new positions of the books, so that

[4] (a) no topic is put on the same shelf as before?

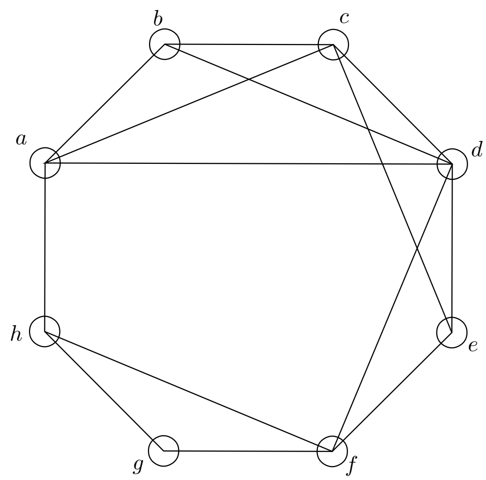
[6] (b) no topic is put to the same shelf as before **and** no book is placed to the same position within a shelf, as before? (For example, if a book was at the second place from the left on a shelf, then it may not be placed to second place on any shelf.)

[10] **4.** How many ways are there to pay 20 dollars in 25-cent, 1-dollar, and 2-dollar coins? Use generating functions! (You don't need to find a numerical answer, a simple formula involving factorials, binomial coefficients, etc. is fine.)

[10] **5.** Solve the following recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 2n + 1 \quad (n \geq 0), \qquad a_0 = 0, a_1 = 0\,.$$

6. Answer the following questions for the graph on the figure. (No explanation needed.)



[1] (a) How many loops does the graph have?

[1] (b) Does the graph have multiple edges?

[2] (c) How many 3-cycles does the graph have? (3-cycles having the same vertices and edges count only once).

[2] (d) How many induced subgraphs does the graph have?

[1] (e) Specify a subgraph that is not induced.

[1] (f) Write down edges of a spanning tree.

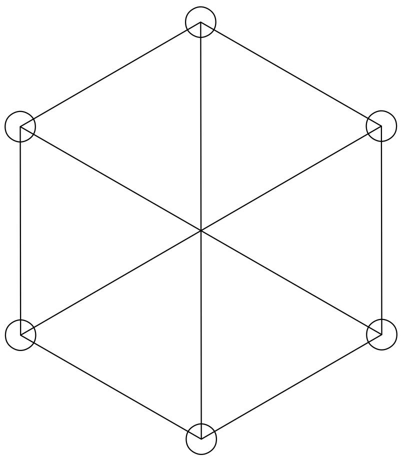
[1] (g) Write down a path of length 3 (that is, with three edges) between vertices a and b.

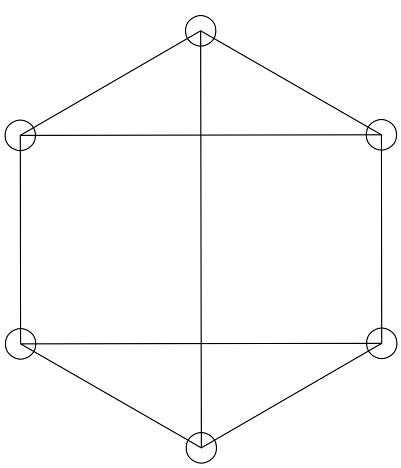
[1] (h) How many edges does the complement of the graph have?

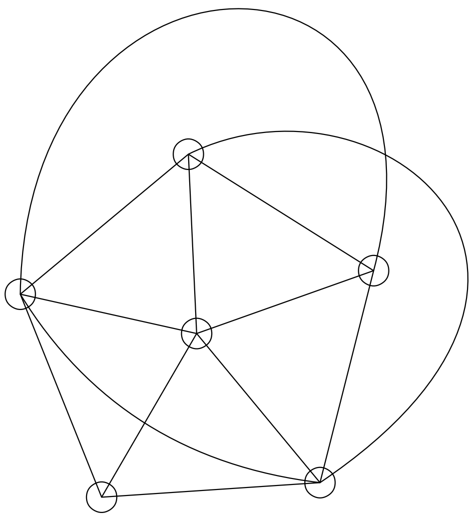
[5] **7.** (a) Recall that for $n \geq 1$ the n-dimensional hypercube is the graph $Q_n = (V, E)$, where $V = \{0, 1\}^n$, and for $x, y \in V$, we have $\{x, y\} \in E$ if and only if x and y differ in exactly one coordinate.
For which integers n does Q_n have an Euler circuit? Explain!

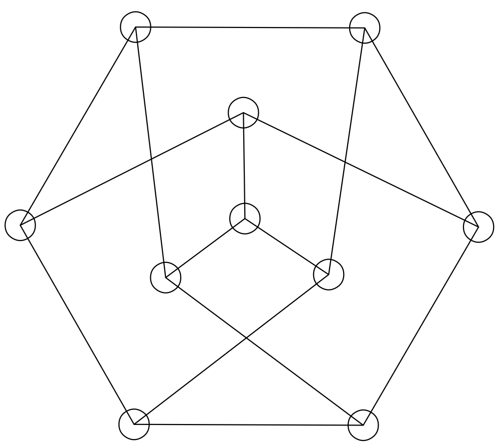
[5] (b) For which integers n does Q_n have a Hamilton cycle? Explain!

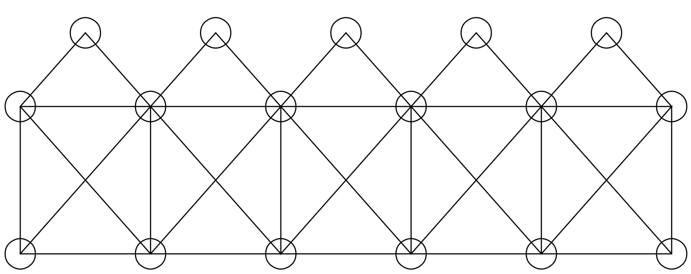
[10] **8.** Are the following graphs planar? If yes, draw the graph without crossing, if not, explain why not. [2 marks for each graph]



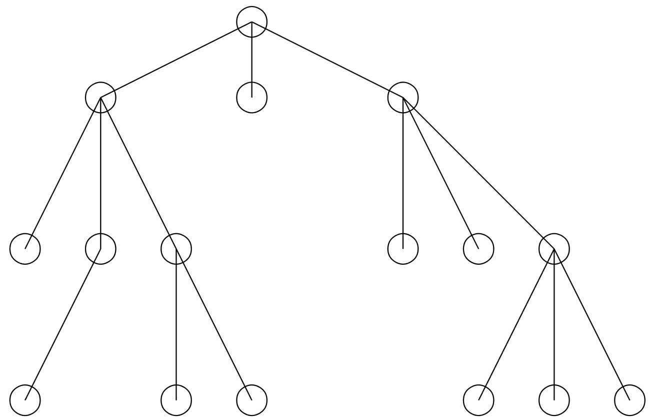








[4] **9.** (a) For the rooted tree on the figure, determine the labels of the vertices according to the universal address system.



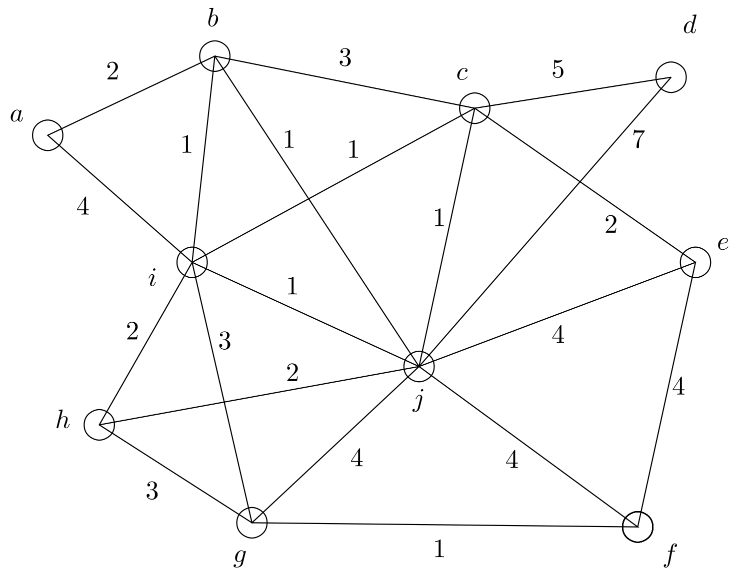
[2] (b) Construct the binary rooted tree for the following arithmetic expression:

$$(2 + 3 * 4)/(5 * (7 + 3) + 14)$$

[2] (c) Use the postorder traversal to write the expression in the postfix notation.

[2] (d) Use the preorder traversal to write the expression in the prefix notation.

[5] **10.** (a) Apply Dijkstra's algorithm to the following undirected graph in order to find the shortest path from a to each of the other vertices. Fill each of these distances in the table below and provide a shortest path from a to f. Explain in detail the first three steps of the algorithm and label the vertices v_0, ..., v_9 according to the algorithm.
(The graph is undirected, which means that for each edge $e = \{x, y\}$ we have $wt(x, y) = wt(y, x) = wt(e)$.)



a	b	c	d	e	f	g	h	i	j

[5] (b) Apply Kruskal's algorithm to the following graph. Determine the minimal possible sum of the weights of the edges of a spanning tree and draw an example of a tree that achieves this. Also, label the edges by $e_1, \ldots, e_9$ in the order you analyze them in the algorithm and explain in detail the first three steps of the algorithm.