Mathematics 158-3

Instructor: R. Russell
Room: C9002
Date: March 14, 2005
Time: 17:30-18:20

Second Midterm

NAME:

STUDENT NUMBER:

Write your name and student number in the blank space above. You have 50 minutes to write the exam, so budget your time accordingly. Write down sufficient details to clearly show work. GOOD LUCK!

ONLY SCIENTIFIC CALCULATORS ARE PERMITTED: Calculators with alphanumeric storage, graphics tools or communication capacity are not allowed.

Problem	**Marks**
1a	
1b	
2a	
2b	
3a	
3b	
4a	
4b	
5a	
5b	
Total	

Problem 1 Evaluate the following integrals or show that they do not exist:

(a) $\int_{-\infty}^{0} \frac{1}{\sqrt{4-x}} dx$

(10+10)

(b) $\int_{1}^{e} x \ln x dx$. SIMPLIFY your answer for this problem.

Problem 2 Find the following:

(a) the average value of the function $f(x) = (2x-1)\sqrt{x^2+x-2}$ over its domain.

(10+10)

(b) $\int e^{\sqrt{x}} dx$

Problem 3 A saw mill built 10 years ago is growing but, due to limited lumber resources, cannot have more than 10,000 employees. Initially, there were 2,100 employees, and now there are 3,000.

(a) Assuming that the growth is logistic, recall that the number of employees $E(t)$ satisfies the differential equation

$$E'(t) = KE(M - E)$$

. Solve this equation to find the general form for $E(t)$.

(10+8)

(b) For this particular problem, find $E(t)$ and use it to calculate the number of employees in the saw mill 90 years from now.

Problem 4 Consider the 3 linear equations $2x+2y-2z=-8$, $-2x+2z=4$, and $x+3y-z=-8$.

(a) Write down the augmented coefficient matrix for this system of equations and form the reduced matrix using elementary row operations.

(12+8)

(b) From part (a), give solutions to the problem, explain (i)if there are none, why not; (ii) if there is one, what it is; and (iii) if there are many, what their explicit form is.

(10)

Problem 5(i) Consider the objective function $z = 3x_1 + 2x_2$ subject to the constraints $x_1 + 2x_2 \leq 3$, $2x_1 - x_2 \leq 2$, $-x_1 + 2x_2 \leq 1$, $x_1, x_2 \geq 0$.

(a) Use a graph to show the feasible region for the LP problem involving maximizing z.

(12)

Problem 5(ii) Suppose that we want to solve for the maximum value.
(b) Write down the simplex tableau for the problem, and find the optimal value using the simplex method. Once you find the answer, explain which point it is in the feasible region in part (a).