Note Title 04/03/2008

Midterm II – Solutions

$\boxed{P_1}$ $$\int 3x \cos(4x)\,dx = 3\int x\cos(4x)\,dx$$

$u = x \Rightarrow du = dx$

$dv = \cos(4x)\,dx \Rightarrow v = \frac{1}{4}\sin(4x)$

$$3\int x\cos(4x)\,dx = 3\cdot\left[x\cdot\frac{1}{4}\sin(4x) - \int\frac{1}{4}\sin(4x)\,dx\right]$$

$$= \frac{3}{4}x\sin(4x) - \frac{3}{4}\underbrace{\int \sin(4x)\,dx}_{= -\frac{1}{4}\cos(4x) + C}$$

$$= \frac{3}{4} x \sin(4x) + \frac{3}{16} \cos(4x) + C$$

$\boxed{P_2}$ (a) $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^2}\,dx$

Compute $\displaystyle\int_1^b \frac{\ln x}{x^2}\,dx$ by integration by parts

$u = \ln x \implies du = \frac{1}{x}\,dx$

$dv = \frac{1}{x^2}\,dx \implies v = -\frac{1}{x}$

$$\int_1^b \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x \Big|_1^b + \int_1^b \frac{1}{x^2}\,dx$$

$$= -\frac{1}{b}\ln b + 1\cdot\ln 1 + \left(-\frac{1}{x}\right)\Big|_1^b$$

$$= -\frac{1}{b}\ln b - \frac{1}{b} + 1$$

Now: $\displaystyle\lim_{b\to\infty}\int_1^b \frac{\ln x}{x^2}\,dx = \lim_{b\to\infty}\left\{-\frac{1}{b}\ln b - \frac{1}{b} + 1\right\} = \boxed{1}$

(b) $\displaystyle\int_2^\infty \frac{1}{\sqrt{2x+1}}\,dx = \lim_{b\to\infty}\int_2^b \frac{1}{\sqrt{2x+1}}\,dx.$

Compute $\displaystyle\int_2^b \frac{1}{\sqrt{2x+1}}\,dx =$

substitution $u = 2x+1$, $du = 2\,dx$

$$= \int_5^{2b+1} u^{-\frac{1}{2}} \frac{1}{2}\,du = \frac{1}{2}\, 2u^{\frac{1}{2}} \Big|_5^{2b+1} = \sqrt{2b+1} - \sqrt{5}$$

$$\Rightarrow \int_2^{\infty} \frac{1}{\sqrt{2x+1}}\,dx = \lim_{b \to \infty} \left\{ \sqrt{2b+1} - \sqrt{5} \right\} = \infty$$

the integral is divergent

$\boxed{P_3}$

$$P = \int_0^t f(x)\, e^{-rx}\,dx$$

$$= \int_0^{10} \underbrace{150\, e^{-0.02x}}_{f(x)}\ \underbrace{e^{-0.08x}}_{e^{-rx}.}\,dx$$

$$= 150 \int_0^{10} e^{-0.1x}\,dx = 150 \frac{1}{-0.1} e^{-0.1x} \Big|_0^{10}$$

$$= -1500 \left(e^{-1} - e^{0} \right)$$

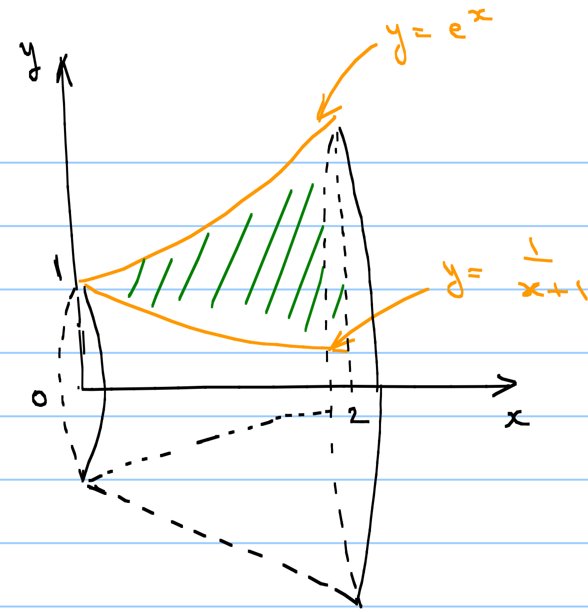
$$= 1500 \left(1 - e^{-1} \right)$$

$$A = e^{rt} P$$

$$= e^{0.08 \cdot 10} \cdot 1500 \left(1 - \frac{1}{e} \right) =$$

$$= 1500\, e^{0.8} \left(1 - e^{-1} \right)$$

$\boxed{P_4}$



Formula:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

For our problem:

$$V = \pi \int_0^2 (e^x)^2 \, dx - \pi \int_0^2 \left(\frac{1}{x+1}\right)^2 dx$$

$$\Rightarrow \quad V = \pi \int_0^2 e^{2x} \, dx - \pi \int_0^2 \frac{1}{(x+1)^2} \, dx$$

$$= \frac{\pi}{2} e^{2x} \Big|_0^2 + \pi \frac{1}{x+1} \Big|_0^2$$

$$= \frac{\pi}{2} \left[e^4 - 1 \right] + \pi \left[\frac{1}{3} - 1 \right]$$

$$= \frac{\pi}{2} e^4 \underbrace{- \frac{\pi}{2} - \frac{2\pi}{3}}_{-\frac{7\pi}{6}} = \frac{\pi}{2} \left[e^4 - \frac{7}{3} \right]$$

$\boxed{P_5}$ (a) $f_x(x,y) = 3 \frac{x+y^2-x}{(x+y^2)^2} = 3 \frac{y^2}{(x+y^2)^2}$

$$f_y(x,y) = 3x \cdot \left(\frac{-1}{(x+y^2)^2} \right) \cdot 2y = \frac{-6xy}{(x+y^2)^2}$$

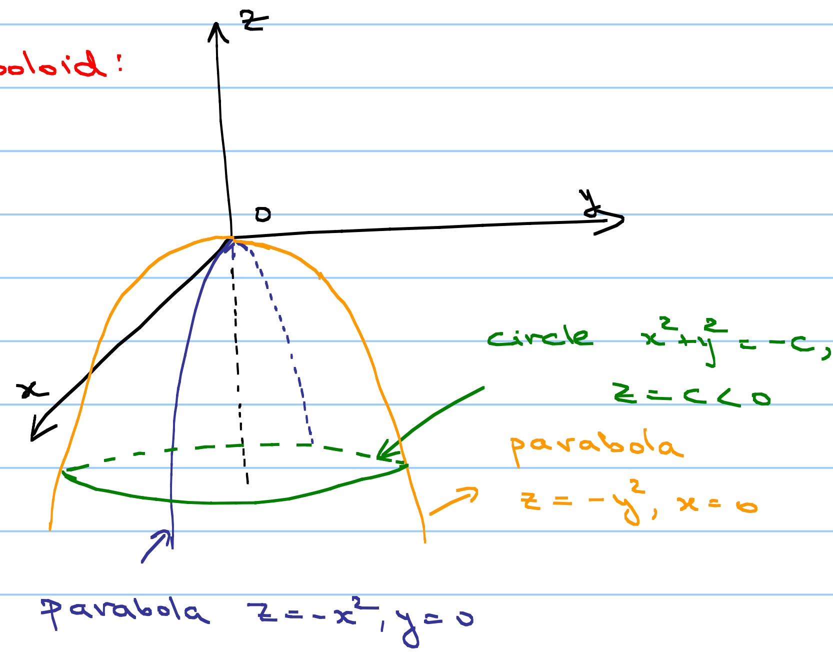
$$f_{xy}(x,y) = 3 \quad \frac{2y\,(x+y^2)^2 - y^2 \cdot 2(x+y^2) \cdot 2y}{(x+y^2)^2}$$

$$= 3 \quad \frac{2y\,(x+y^2) - 4y^3}{(x+y^2)^2}$$

$$= 3 \quad \frac{2xy - 2y^3}{(x+y^2)^2} = \frac{6y\,(x-y^2)}{(x+y^2)^2}.$$

(b) Observe that $z \le 0$, also observe that the graph passes through $(0,0,0)$

paraboloid:



Traces:

- intersection with $z = \text{const} < 0$

  $x^2 + y^2 = -c > 0 \Rightarrow$ circle

  with center on the z-axis

  with radius $\sqrt{-c}$

- intersection with the yz-plane

  $x = 0 \Rightarrow z = -y^2$ parabola

- intersection with the xz-plane

  $y = 0 \Rightarrow z = -x^2$ also a parabola