

Mathematics 158-3

Instructor: R. Russell
Room: C9002

Date: February 7, 2005
Time: 17:30-18:20

First Midterm

NAME:

STUDENT NUMBER:

Write your name and student number in the blank space above. You have 50 minutes to write the exam, so budget your time accordingly. Write down sufficient details to clearly show work. GOOD LUCK!

NO CALCULATORS ALLOWED

Helpful formulas: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

Problem	Marks
1a	
1b	
2	
3a	
3b	
4a	
4b	
4c	
4d	
5	
6a	
6b	
Total	

(8+8) **Problem 1** Evaluate the following integrals:

(a) $\int \frac{\ln x^2}{x} dx$

(b) $\int \frac{(x+3x^2)}{e^{2x^2+4x^3}} dx$

- (12) **Problem 2** Find the bounded area between the graphs of the two functions $f(x) = 1 - 3x$ and $g(x) = 2 - x^2$. Draw a picture to help check your answer.

Problem 3 Find the following definite integrals:

(a) $\int_1^5 \frac{3x+2x^3}{x^2} dx$

(8+8)

(b) $\int_{-1}^0 x^3 \sqrt{x+3} dx$

Problem 4(i) Suppose that we want to approximate the definite integral $I = \int_0^2 (10 - 2x^2) dx$.

(a) First, find I exactly using the antiderivative.

(6+8)

(b) Using $n = 4$ subintervals, draw a picture to show how the trapezoidal rule approximates the area given by I , and find the approximation T_4 . You do NOT need to simplify your answer.

Problem 4(ii) Continued:

(c) Find the upper sum \bar{S}_n to I , where \bar{S}_n is the approximation which uses n subintervals (here, n is arbitrary and remember that we approximate the area for each slice using a rectangle).

(8+6)

(d) Take $\lim_{n \rightarrow \infty} \bar{S}_n$ to check your value of I in part (a).

(12) **Problem 5** Solve the differential equation $y' = \frac{2t^2 - 6t}{-2t^3 + 9t^2 + 1}$, using the initial condition $y(0) = \frac{1}{3}$.

Problem 6 Suppose that the demand equation for a product is $p = .01q^2 - 1.1q + 30$ where p is the price p and quantity q are measured in suitable units. Suppose that the corresponding supply equation is $p = .01q^2 + q + 8$.

(a) Find the consumers' surplus under market equilibrium. Use a graph to justify how you obtain your answer.

(10+6)

(b) Find the corresponding producers' surplus.