Mathematics 158-3

Instructor: R. Russell **Date:** April 13, 2005
Room: C9002 **Time:** 19:00-22:00

FINAL EXAM

SURNAME: **GIVEN NAME:**
STUDENT No.: **SIGNATURE:**

Write your name and student number in the blank space above. Do not start the exam until instructed to do so.

Make certain that you have all **ELEVEN** *pages of the exam. You have three hours, so budget your time accordingly.*

Write down sufficient details to clearly show work. **You need to simplify answers when specifically requested to do so**.

ONLY SCIENTIFIC CALCULATORS ARE PERMITTED: Calculators with alphanumeric storage, graphics tools or communication capacity are not allowed.

Problem	Possible Marks	Actual Marks
P1	32	
P2	10	
P3	14	
P4	10	
P5	12	
P6	16	
P7	20	
P8	18	
P9	18	
Total	150	

Problem P1 Evaluate the following integrals:

(a) $\int (x-\pi)e^{x-\pi}dx$

(8 each)

(b) $\int \frac{10x-2x^2}{(x-1)^2(x+3)}dx.$

Problem P1 cont'd

(c) $\int_0^1 \frac{x}{x+2} dx$ Simplify you answer.

(8 each)

(d) Using the definition of an improper integral, find $\int_1^\infty \frac{x^2}{(x^3+2)^{5/2}} dx$ if it exists. Simplify your answer.

(10) **Problem P2** Find the area between the two curves $y = \sqrt{5x+6}$ and $y = |x|$ and the lines $x = -1$ and $x = 2$. Draw a graph to motivate your answer.

Problem P3

(a) Use the trapezoidal rule with 4 subintervals to approximate the integral $I = \int_1^3 \frac{1}{x} dx$. Your final answer should be in terms of a sum of numbers, which you do not have to simplify.

(8+6)

(b) Draw a picture to show how the trapezoidal rule is actually approximating the integral, and based upon this picture, state whether this trapezoidal rule approximation is larger than or smaller than the exact integral I if it is possible to tell.

Problem P4

(10) A certain mining town in the interior of British Columbia had a population of 45,000 in 1970 and 32,000 in 1990. Assuming that $P(t)$, the rate of population decline, is proportional to the size of the population, what differential equation represents this rate of decline for $P(t)$? What is the expected population in 2010?

Problem P5

(a) Solve the differential equation $2\frac{dy}{dx} = y(4-y)$ with $y > 4$ with the initial condition $y = 5$ when $x = 0$.

(8+4)

(b) Explain what the graph of y looks like for $x > 0$ without using the explicit solution in (a), but instead *using only the differential equation and initial condition*. Hint: What is the derivative of y?

Problem P6 Consider the linear programming problem (LP) of minimizing $w = 6y_1 + 2y_2$ subject to $3y_1 + 2y_2 \geq 12$, $y_1 - y_2 \geq 4$, $y_1, y_2 \geq 0$.

(a) What is the dual problem?

(8+8)

(b) Using a graph, show the feasible region for the LP in (a). Simply by drawing lines corresponding to different fixed values of the objective function, at what point can you say that the maximum value must occur?

Problem P7 Consider the LP of finding the maximum of $w = 2x - 3y + 4z$ subject to the constraints $x + y - 2z \geq -3$, $x + z \geq 5$ and $-x - 2y + z \leq -2$, and $x, y, z \geq 0$.

(a) Set up simplex tableau for it, clearly labelling all variables variables in the tableau and mentioning what *types* of variables they are.

(12+8)

(b) Carry out *the first full step* of the simplex method on this problem, noting which variables are switched.

Problem P8 Suppose that two dice are tossed. Consider the events $E_1 = \{\text{first die is } \leq 3\}, E_2 = \{\text{the sum is } 11\}$, $E_3 = \{\text{both dice are odd}\}$.

(a) Which events are mutually exclusive?

(6+12)

(b) Which events are dependent? Justify your answers by finding appropriate information needed to apply the definition of "dependent".

Problem P9

(a) If 5 cards are drawn without replacement from a standard 52 card deck, what is the probability that there are exactly 3 of a kind (for example 3 queens, or 3 aces)? Explain.

(8+10)

(b) Suppose that three equally populated regions in Vancouver are polled for their opinions about capital punishment. In Region A, 30% support capital punishment, 30% oppose it, and 40% have no opinion. In Region B, 20% support capital punishment, 40% oppose it, and 40% have no opinion. In Region C, 60% support capital punishment, 10% oppose it, and 30% have no opinion. If a person chosen at random supports capital punishment, what is the probability that they are from Region A? Explain.