SIMON FRASER UNIVERSITY

MATH 157 Quiz 3

Section D100

November 26, 2007

Time: 11:35 – 12:15 (40 min)

Last Name ______________________

Given Name(s) ______________________

Student # ______________________

SFU email ID ______________________

Student signature

INSTRUCTIONS

1. **Do not open this booklet until told to do so!**
2. This exam has 4 questions on 4 pages excluding this cover page. Please check to make sure your exam is complete.
3. **Calculators are not allowed. No electronic devices may be within reach of a student.**
4. Write your full name, student number and SFU email ID on the cover page.
5. Please read the questions carefully, and make sure you understand what you are asked to do!
6. Please write with a black or blue **pen**.
7. Use the reverse side of the **previous page** if you need more room for your rough work.
8. **You may lose marks if your explanations are incomplete, missing, or poorly presented.**
9. You may attempt the questions in any order.
10. **You must stop writing immediately when asked to do so!**

Question	Score
1	/10
2	/10
3	/12
4	/8
Total	/40

1. **TRUE or FALSE**. No explanation is required for your choice.

[2] (a) If $f(c) = 0$, then f must have a relative extremum at c.

TRUE: ☐ FALSE: ☐

[2] (b) For the function $f(x) = \dfrac{1}{x}$ we have $f(2) = \dfrac{1}{2}$ and $f(-1) = -1$. The mean value theorem hence tells us that there is a point c, $-1 < c < 2$, such that

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)} = \frac{3/2}{3} = \frac{1}{2}.$$

TRUE: ☐ FALSE: ☐

[2] (c) On the interval $(-1, 2)$, the function $f(x) = x^4$ has an absolute maximum and an absolute minimum.

TRUE: ☐ FALSE: ☐

[2] (d) If $f''(c) = 0$, then $(c, f(c))$ must be a point of inflection.

TRUE: ☐ FALSE: ☐

[2] (e) A function f defined on the interval $(-2, 2)$, with exactly one relative maximum, must have an absolute maximum.

TRUE: ☐ FALSE: ☐

2. **Differentiation**.

[4] (a) The function $y(x)$ is given implicitly by the equation

$$x + \sin(y) = x^2 y^3.$$

Note that $y(0) = 0$. Find $y'(0)$, i.e., evaluate the derivative $\frac{dy}{dx}$ at $x = 0$.

[3] (b) Use Newton's method to estimate $\sqrt[5]{40}$. Use 2 as your initial guess, and perform the first step of the method only.

[3] (c) Use linear approximation to estimate the value of $\sqrt[5]{40}$, only making use of $\sqrt[5]{32} = 2$. Compare your linear approximation to the result obtained by Newton's method in part (b) of this question.

SHOW YOUR WORK

3. **Profit Maximization**. Remember the Hummerator Mountain Bike from Quiz 2? The bicycle company is now under new management. The new manager, George O'Brien, estimates the monthly cost function $C(x)$ to produce the Hummerator and the monthly demand function as

$$\text{Cost: } C(x) = 400 + 80x + x^2, \qquad \text{Demand: } p = 400 - x,$$

where x is the number of bicycles produced and sold per month, and cost and price p are measured in Dollars.

[1] (a) What is $R(x)$, the monthly revenue as a function of x?

[1] (b) What is $P(x)$,the monthly **PROFIT** as a function of x?

[4] (c) Determine the level of output at which profit is maximized.

[1] (d) Determine the price at which maximum profit occurs.

[5] (e) If, as part of a "green inititative", the government provided a subsidy of $\$20$ per bike to the company, then

- what is the new cost function?
- what is the new price for profit maximization?
- how much of this subsidy effectively goes to the company, how much to the consumer?

4. Elasticity The weekly demand function for a new book *"How to Pass Math157 Without Paying Attention in Class"* is given by $q = 300 - 2p$ where q is the quantity and p is the price.

[4] (a) Find the elasticity of demand, E.

[2] (b) At what price will the demand have unit elasticity?

[2] (c) At what price will the authors realize maximum revenue?

SHOW YOUR WORK