Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157-3, Summer 2006
Midterm 2
July 10th, 2006, 11:30 – 12:20

Last Name (please print):	KEY
First Name (please print):	
Student Number:	
Instructor:	P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains ~~7~~ 6 pages with a total of 5 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table!	
Question	**Marks**
1	**/8**
2	**/12**
3	**/4**
4	**/10**
5	**/6**
Total	**/40**

1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. Every correct answer will receive **1**. **[8 marks]**

a) F The product rule for $f(x) = g(x)h(x)$ says that $f'(x) = g(x)h(x) + g(x)'h(x)'$.

b) F $\lim_{x\to\infty} \frac{\sin x}{x} = 1$.

c) T If $P(q)$ describes the profit function for producing and selling q number of a certain product, then $\overline{P}'(q)$ is the marginal average profit.

d) T Given $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

e) T Given a differentiable function *f*, if $f'(x) < 0$ for all *x*, then *f* is decreasing everywhere.

f) F Critical numbers *c* for a function *f* are only those numbers in the domain of *f* such that $f'(c) = 0$.

g) T Given a twice differentiable function *f* on the interval *I* with $c \in I$, if $f'(c) = 0$ and $f''(c) > 0$, then *c* is the location of a relative minimum.

h) F If $\lim_{x\to\infty} f(x) = 1$, then $y = 1$ is a vertical asymptote for the graph of *f*.

2. Find the indicated derivative for the following functions: **[3 marks each = 12 marks]**

a) $y = \dfrac{x^2 - 3}{\tan(x) + 1}$, $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2x(\tan x + 1) - (x^2 - 3)\sec^2 x}{(\tan x + 1)^2}$$

b) $f(x) = \ln(\sin^2(x^4 + 1))$, $f'(x)$

$$f'(x) = \frac{2\sin(x^4+1)\cos(x^4+1)\,4x^3}{\sin^2(x^4+1)}$$

c) $g(x) = x^{3/2}e^x$, $g'(0)$

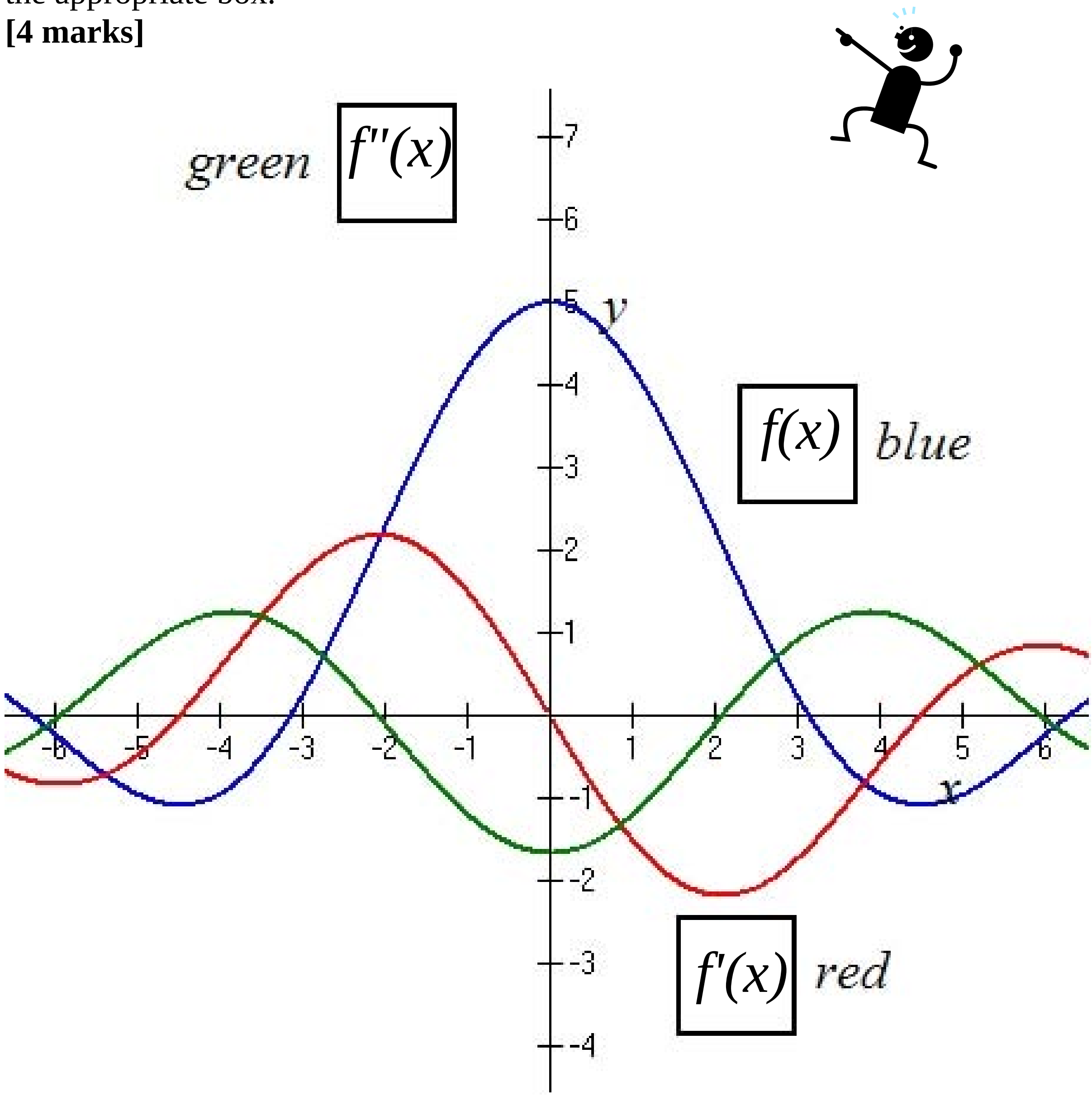
$$g'(x) = \frac{3}{2}x^{\frac{1}{2}}e^x + x^{3/2}e^x$$

$$g'(0) = 0$$

d) $y = 3e^{\pi - 2} + \sin 4$, y'

$$y' = 0$$

3. Below are the graphs of a function and its first and second derivatives defined for all real numbers. Label the curves by placing $f(x)$, $f'(x)$, and $f''(x)$ into the appropriate box.
[4 marks]



4. Given the curve $y = \frac{x^2}{\sqrt{x+1}}$ and its derivatives $y' = \frac{3x^2+4x}{2(x+1)^{3/2}}$ and $y'' = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$.

a) Find the domain of y. **[2 marks]**

$D = (-1, \infty)$

b) Find the intervals of increase and decrease of the curve. **[3 marks]**

$y' = 0 \Rightarrow 3x^2 + 4x = 0 \Rightarrow x = 0, -\frac{4}{3}$ reject

$y' = \text{DNE}$ no such x

	-1	0
y'	$-$	$+$
y	decr.	incr.

y is increasing on $(0, \infty)$

" " decreasing " $(-1, 0)$

c) Find all minima and maxima points. **[2 marks]**

y only has a minimum at $x = 0$

by the first derivative test:

$(0, 0)$

d) Find the intervals of concave up and concave down of the curve. **[3 marks]**

$y'' = 0 \Rightarrow 3x^2 + 8x + 8 = 0$ impossible

$y'' = \text{DNE}$ no such x

So, y is either always c.u. or always c.d.

Since $y''(0) > 0$ we conclude y is always c.u.

5. An apple orchard owner has determined that if 30 trees are planted per acre, the yield is 600 apples per tree. For each tree per acre that is added beyond 30, the yield is reduced by 15 apples per tree. **[6 marks]**

a) Write an equation for the yield function.

Let x be the number of trees added.

$f(x) = \text{trees} \times \text{apples}$

$f(x) = (30 + x)(600 - 15x)$

$= -15x^2 + 150x + 18{,}000$

b) Find the domain of the yield function.

$D = [0, \infty)$

c) Find the number of trees per acre that should be planted to yield the maximum crop.

$f'(x) = -30x + 150$

$-30x + 150 = 0$

$\Rightarrow x = 5$

		5	
f'	$+$		$-$
f	incr.		decr.

$\Downarrow$

max by 1st deriv. test.

So, $30 + 5 = 35$ trees / acre should be planted.

OR

$f(x) = -15(x^2 - 10x) + 18{,}000$

$= -15(x-5)^2 + 18{,}375$

This is a parabola with vertex $(5, 18{,}375)$ that opens downward, i.e, the max is the vertex.

So, $30 + 5 = 35$ trees / acre should be planted.