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(FAMILY NAME) (GIVEN NAME) (SFU ID)

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Simon Fraser University
Department of Mathematics
Midterm Examination 2
MATH 157
13 March 2006 11:30–12:20

Question	Score	Maximum
1		8
2		6
3		4
4		10
Total		28

- Please ensure that you sign your exam above to certify your identity. Unsigned exams will not be marked.
- Only scientific, non-programmable calculators with no differentiation nor integration nor graphing capability are allowed in exams.
- The duration of this exam is 50 minutes.
- DO NOT OPEN this test booklet until told to do so.
- Please check that you have all 6 pages of the exam.
- Do ALL your work in this test booklet. You may use the backside of each page for scrap work.
- The value of each question is shown on the left margins.

1. Find the indicated derivative for the following functions. Do not simplify your answer for 1a. and 1b.

[2] (a) $f(x) = \frac{\tan(x)+\frac{3}{4}}{\sqrt[3]{\pi-e^2}}$, $f''(x) = ?$

We have that

$$f'(x) = \frac{1}{\sqrt[3]{\pi - e^2}} \sec^2(x)$$

and hence

$$f''(x) = \frac{1}{\sqrt[3]{\pi - e^2}} \cdot 2\sec(x) \cdot \sec(x)\tan(x).$$

[2] (b) $y = \frac{x-1}{\cos(x)+e^x}$, $\frac{dy}{dx} = ?$

We have that

$$\frac{dy}{dx} = \frac{(\cos(x) + e^x) + (x-1)(-\sin(x) + e^x)}{(\cos(x) + e^x)^2}.$$

[2] (c) $f(x) = 3^x \log_2(x^4 + 1)$, $f'(0) = ?$

We have that

$$f'(x) = 3^x \ln 3 \cdot \log_2(x^4+1) + 3^x \cdot \frac{1}{(x^4+1)\ln 2} \cdot 4x^3.$$

Hence, $f'(0) = 0$.

[2] (d) $g(1) = 0, g(2) = 1, h(2) = 2, g'(1) = 1, g'(2) = -3, h'(2) = 1, f(x) = g(h(x))$, $f'(2) = ?$

By the chain rule, we have that $f'(x) = g'(h(x)) \cdot h'(x)$. Hence, $f'(2) = g'(h(2)) \cdot h'(2) = g'(2) \cdot 1 = -3$.

2. Let $f(x) = \frac{2(x+2)}{(x-1)(x-3)^2}$.

[2] (a) Determine the values of x such that $f(x) = 0$ or is not defined.

The values of x such that $f(x) = 0$ or is not defined are $x = -2, 1, 3$.

[4] (b) Determine the intervals on which $f(x) > 0$ and the intervals on which $f(x) < 0$ by filling out the sign of $(x+2)$, $(x-1)$, $(x-3)^2$, and $f(x)$ in the table below.

	$(x+2)$	$(x-1)$	$(x-3)^2$	$f(x)$
$(-\infty, -2)$				
$(-2, 1)$				
$(1, 3)$				
$(3, \infty)$				

The completed table is given below.

	$(x+2)$	$(x-1)$	$(x-3)^2$	$f(x)$
$(-\infty, -2)$	$-$	$-$	$+$	$+$
$(-2, 1)$	$+$	$-$	$+$	$-$
$(1, 3)$	$+$	$+$	$+$	$+$
$(3, \infty)$	$+$	$+$	$+$	$+$

[4] **3.** Suppose the demand of a certain product as a function of price is given by $x = 12 - p^2$. Calculate the revenue R as a function of price and determine its absolute extrema on $[1, 3]$.

We have that $R(p) = xp = (6 - p^2)p = 12p - p^3$. Now $R'(p) = 12 - 3p^2$. The critical numbers of $R(p)$ in $[1, 3]$ are $p = 2$. We evaluate $R(p)$ at the relevant critical numbers and endpoints to get

$$R(1) = 11, R(2) = 16, R(3) = 9.$$

Hence, the absolute minimum of $R(p)$ on $[1, 3]$ is 9 and the absolute maximum of $R(p)$ on $[1, 3]$ is 16.

4. Suppose $f(x)$ is a continuous function on $\mathbb{R}$ such that

1. $f(-2) = 1, f(0) = 6, f(1) = 5, f(2) = 4, f(3) = 2$
2. $\lim_{x\to-\infty} f(x) = 0$
3. $\lim_{x\to\infty} f(x) = 4$
4. Differentiable everywhere except at $x = 3$
5. $f'(0) = f'(2) = 0$
6. $f'(x) > 0$ on $(-\infty, 0)$ and $(3, \infty)$
7. $f'(x) < 0$ on $(0, 2)$ and $(2, 3)$
8. $\lim_{x\to3^-} f'(x) = -\infty$, $\lim_{x\to3^+} f'(x) = \infty$
9. $f''(x) > 0$ on $(-\infty, -2)$ and $(1, 2)$
10. $f''(x) < 0$ on $(-2, 1)$, $(2, 3)$ and $(3, \infty)$

[2] (a) Determine the intervals of increase/decrease and concave up/down by filling out the following table.

	$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
$f'(x)$						
$f''(x)$						
increase/decrease						
concave up/down						

The completed table is given below.

	$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
$f'(x)$	$+$	$+$	$-$	$-$	$-$	$+$
$f''(x)$	$+$	$-$	$-$	$+$	$-$	$-$
increase/decrease	inc	inc	dec	dec	dec	inc
concave up/down	up	down	down	up	down	down

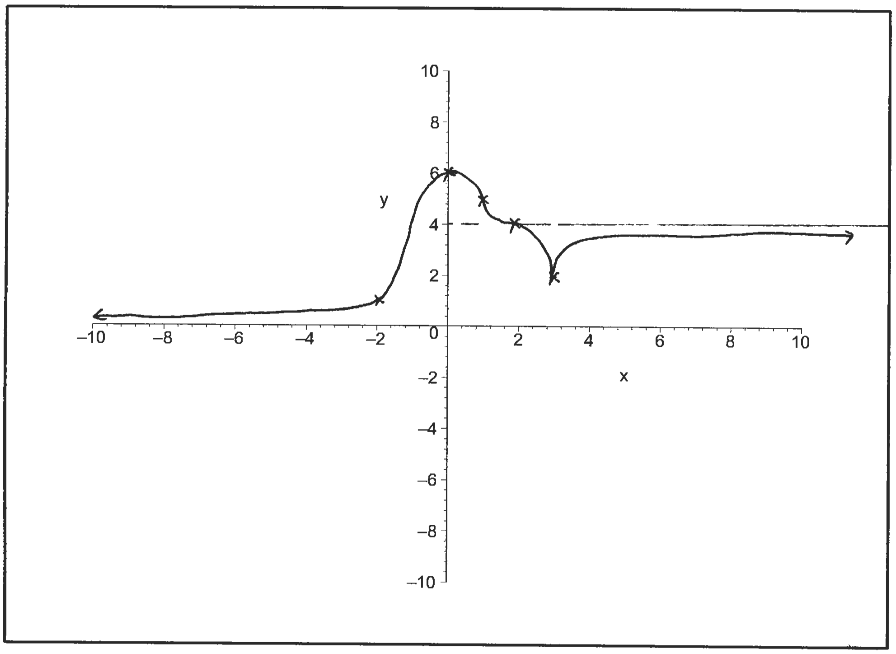
[2] (b) Determine the values of c such that $f(c)$ is a relative extrema.

The critical numbers occur at $c = 0, 2, 3$. By the first derivative test, relative extrema occur at $c = 0, 3$.

[2] (c) Determine the values of c such that f has an inflection point at c.

The values of c such that $f''(c) = 0$ or $f''(c)$ does not exist are $c = -2, 1, 2, 3$. By looking at the sign of $f''(x)$ to the left and right of each point above, we see that at $c = -2, 1, 2$, f has inflection points.

[4] (d) Sketch the graph of $y = f(x)$.



10
8
6
y
4
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2
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x