Version "A"

SIMON FRASER UNIVERSITY

MATH 157 Quiz 2

Section D100

October 29, 2007

Time: 11:35 – 12:10

Last Name _____

Given Name(s) _____

Student # _____

SFU email ID _____

Student signature

INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. This exam has 4 questions on 4 pages excluding this cover page. Please check to make sure your exam is complete.
3. **Calculators are not allowed. No electronic devices may be within reach of a student.**
4. Write your full name, student number and SFU email ID on the cover page.
5. Please read the questions carefully, and make sure you understand what you are asked to do!
6. Please write with a black or blue **pen**.
7. Use the reverse side of the **previous page** if you need more room for your rough work.
8. **You may lose marks if your explanations are incomplete, missing, or poorly presented.**
9. You may attempt the questions in any order.
10. **You must stop writing immediately when asked to do so!**

Question	Score
1	/10
2	/10
3	/10
4	/10
Total	/40

A

1. Agony of Choice. The following questions are multiple choice. No explanation is required for your answer.

[2] (a) $f(x) = \ln(1-x^2)$, where $-1 < x < +1$. Then $f'(x) =$

$\ln(1-x^2) = \ln(1-x) + \ln(1+x)$

A: ☐ $\frac{1}{1+x} + \frac{1}{1-x}$, B: ☐ $\frac{1}{1-x^2}$,

C: ☑ $\frac{1}{1+x} - \frac{1}{1-x}$, D: ☐ $\frac{2}{1-x^2}$,

E: ☐ None of the above

[2] (b) $f(x) = x^{\frac{3}{4}}$. Then $f'(x) =$

A: ☐ $x^{-\frac{1}{4}}$, B: ☐ $\frac{3}{4}x^{\frac{2}{4}}$, C: ☐ $x^{\frac{1}{4}}$, D: ☑ $\frac{3}{4}x^{-\frac{1}{4}}$,

E: ☐ None of the above

[2] (c) $f(x) = xe^{-x}$. Then $f'(x) =$

PRODUCT RULE

A: ☐ x^2e^{-x-1}, B: ☑ $e^{-x} - xe^{-x}$, C: ☐ e^{-x}, D: ☐ $e^x - xe^x$,

E: ☐ None of the above

[2] (d) $f(x) = \sin^2(x)$. Then $f'(x) =$ $2\sin(x)\cos(x)$ BY CHAIN RULE

A: ☐ $1 - \cos^2(x)$, B: ☐ $2\sin(x)$, C: ☐ 0, D: ☐ $-2\sin(x)\cos(x)$,

E: ☑ None of the above

[2] (e) $f(x) = \ln\sqrt{1+x^2}$. Then $f'(x) =$

A: ☐ $\frac{1}{\sqrt{1+x^2}}$, B: ☑ $\frac{x}{1+x^2}$, C: ☐ $\frac{2x}{\sqrt{1+x^2}}$, D: ☐ $\frac{1}{2(1+x^2)}$,

E: ☐ None of the above

$f = \frac{1}{2}\ln(1+x^2)$ $f' = \frac{1}{2}\frac{1}{1+x^2}2x$

CHAIN RULE

2. Critical situation. $f(x) = x^4 - 4x^3 - 8x^2 + 12$.

[2] (a) Compute $f'(x)$, and $f''(x)$.

$f'(x) = 4x^3 - 12x^2 - 16x = 4x(x^2 - 3x - 4)$

$f''(x) = 12x^2 - 24x - 16$

[4] (b) Find the critical numbers of f. Hint: There are three of them, all integers!

$f'(x) = 4x(x-4)(x+1)$

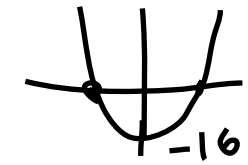
$x = -1 \qquad x = 0 \qquad x = +4$

[2] (c) Which of those critical points are the location of a relative minimum? Check the second derivative to **justify** your answer!

x	-1	0	4
$f''(x)$	20	-16	80

Relative MINIMUM AT $x = -1$ & $x = 4$

[2] (d) Does f have any turning points? If **yes**, how many (you do not have to compute them, but you may), if **no**, explain why not!



has two intersects with x AXIS, CHANGES SIGN

SHOW YOUR WORK

$\Rightarrow$ 2 TURNING POINTS

$12x^2 - 24x - 16 = 0$

$x^2 - 2x - \frac{4}{3} = 0$

$x = 1 \pm \sqrt{1 + 4/3} = 1 \pm \sqrt{\frac{7}{3}}$

3. Bicycle quest for profit. Remember the Exhilarator Mountain Bike from Quiz 1? Well, despite your success in finding the profit function, the "Exhilarator" was a commercial flop. So the bike company has decided to produce a commuter bike, the "Hummerator" (do you think, perhaps they should hire a marketing firm to come up with better names?). The production manager, Peter Griffin, estimates the monthly cost function $C(x)$ to produce the Hummerator and the monthly demand function as

$$\text{Cost: } C(x) = 900 + 200x + \frac{x^3}{3000}, \qquad \text{Demand: } p = 450 - \frac{x}{2},$$

where x is the number of bicycles produced and sold per month, and cost and price p are measured in Dollars.

[2] (a) What is $R(x)$, the monthly revenue as a function of x? (Note, that x depends on the price via the demand function!)

$R(x) = xp = x\left(450 - \frac{x}{2}\right)$

[2] (b) What is $W(x)$,the monthly **PROFIT** as a function of x?

$W(x) = R - C = -\frac{x^3}{3000} - \frac{x^2}{2} + 250x - 900$

[6] (c) Peter Griffin suggests a production level of $x = 100$, but is not sure whether this is a good choice. He has been attending Math 157 lectures, but spent most of the lecture time talking to his friends, so he missed out on all the goodies that would help him figure this out. Obviously, the guy needs help - **your help!**
As someone who has paid attention in class,

- explain to him in one short sentence what marginal profit is;
- compute for him the marginal profit as a function of x;
- based on marginal profit, advise him on whether it would be a good idea to increase or decrease production from the monthly level of 100.

Thank you! Now make sure that you write all of this down on your exam paper (using pen) - chances are that if Peter hasn't paid attention in class, he may not have paid any attention to your patient explanations!

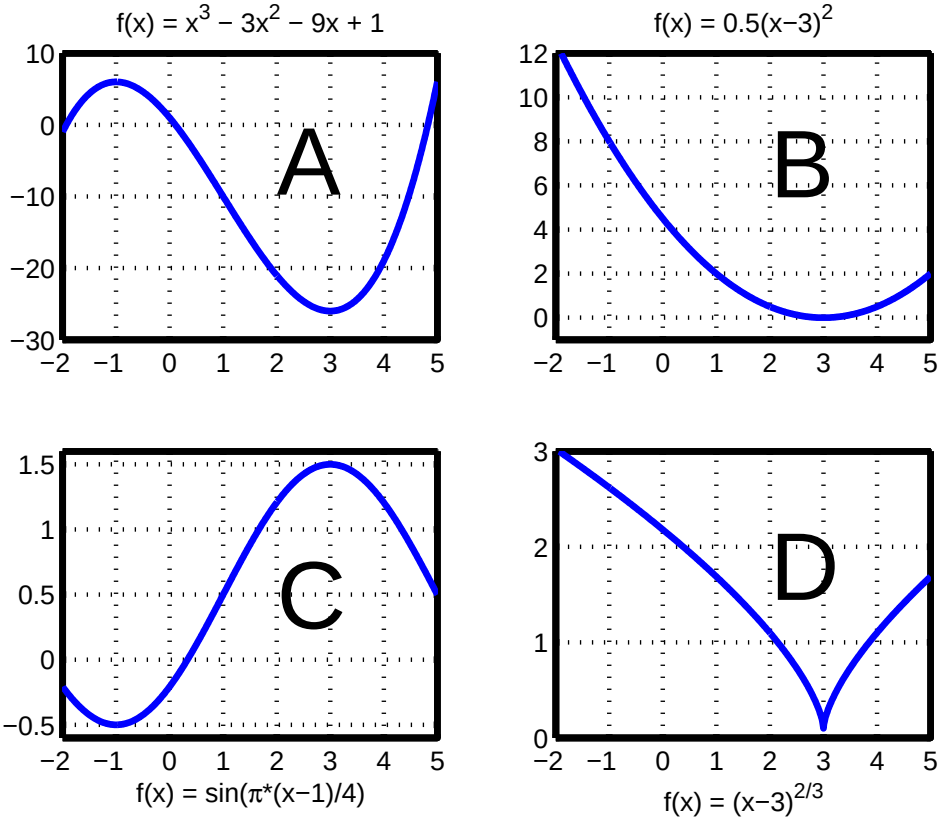
- MARGINAL PROFIT = ADDITIONAL PROFIT BY MAKING & SELLING ONE MORE UNIT ≈ DERIVATIVE OF THE PROFIT FUNCTION

SHOW YOUR WORK

- $W'(x) = -\frac{x^2}{1000} - x + 250$
- $W'(100) = -10 - 100 + 250 = 140 > 0$

INCREASE PRODUCTION!

4. A picture worth a thousand words - actually ten points. Below you have graphs of four functions - A, B, C and D. Check the box by the letter, if the corresponding function has the property mentioned! No explanation required. Caution: More than one picture may fit the description!



[2] (a) The function has a critical point at $x = 3$.
A ☑ B ☑ C ☑ D ☑ None of them ☐

[2] (b) The function has a relative minimum at $x = -1$.
A ☐ B ☐ C ☑ D ☐ None of them ☐

[2] (c) The function has a point where its derivative does not exist.
A ☐ B ☐ C ☐ D ☑ None of them ☐

[2] (d) The function has a turning point at $x = +1$.
A ☑ B ☐ C ☑ D ☐ None of them ☐

[2] (e) The function is concave upward (second derivative > 0) on the interval $[1, 5]$.
A ☑ B ☑ C ☐ D ☐ None of them ☐