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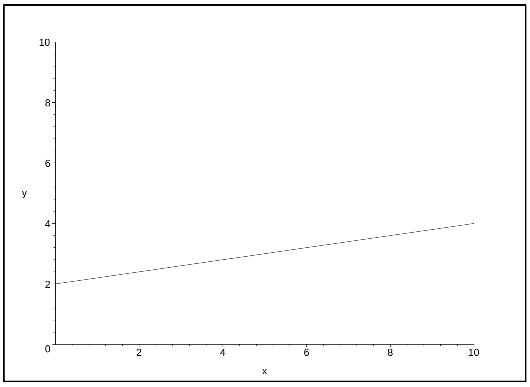
Simon Fraser University
Department of Mathematics
Midterm Examination 1
MATH 157
6 February 2006 11:30–12:20

Question	Score	Maximum
1		6
2		4
3		6
4		4
5		6
6		6
Total		32

- Please ensure that you sign your exam above to certify your identity. Unsigned exams will not be marked.
- Only scientific, non-programmable calculators with no differentiation nor integration nor graphing capability are allowed in exams.
- The duration of this exam is 50 minutes.
- DO NOT OPEN this test booklet until told to do so.
- Please check that you have all 6 pages of the exam.
- Do ALL your work in this test booklet. You may use the backside of each page for scrap work.
- The value of each question is shown on the left margins.

1. The following is a graph of a cost function $C(x)$ as a function of the number of items produced.



[2] (a) What is the fixed cost of production?

The fixed cost is given by the y intercept which is 2.

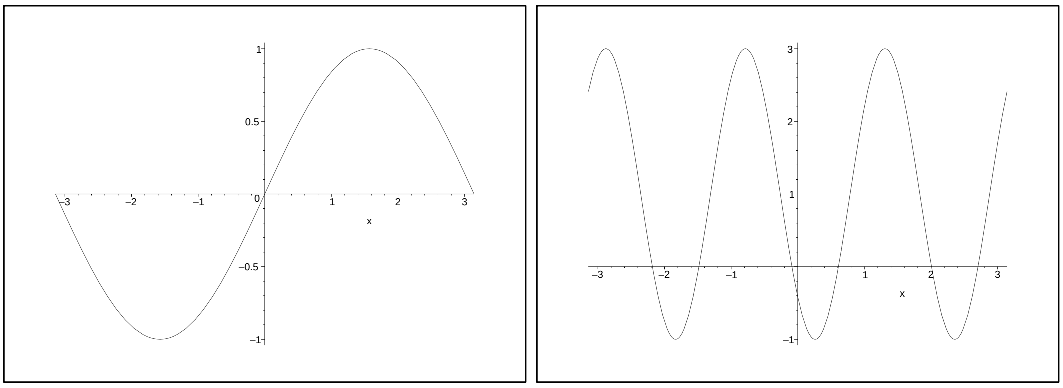
[2] (b) What is the marginal cost of production?

The marginal cost is given by the slope which is 0.2.

[2] (c) Determine the cost function $C(x)$.

The cost function is given by $C(x) = 0.2x + 2$

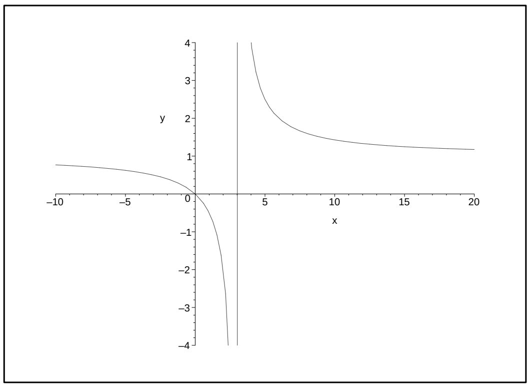
[2] **2.** (a) Circle the graph which best represents the function $y = 2\sin 3(x - \frac{\pi}{4}) + 1$?



Answer: the graph on the right is the correct picture.

[2] (b) Which of the following functions is represented by the graph below? (A) $f(x) = 1/x$, (B) $f(x) = 1/(x-3)$, or (C) $f(x) = x/(x-3)$

Answer: (C)



3. Determine the following limits, if they exist.

[2] (a) $\lim_{x\to-\infty}\frac{6/x+5x^3}{3+2x^2}$

$\lim_{x\to-\infty}\frac{6/x+5x^3}{3+2x^2}=\lim_{x\to-\infty}\frac{6/x^2+5x}{3/x^2+2}=-\infty$

[2] (b) $\lim_{x\to 2^-}\frac{x-2}{|x-2|}$

$\lim_{x\to 2^-}\frac{x-2}{|x-2|}=\lim_{x\to 2^-}\frac{x-2}{-(x-2)}=-1$. We use that fact that if $x<2$ then $|x-2|=-(x-2)$.

[2] (c) $\lim_{x\to\infty}e^{\frac{2x^2+1}{3x^2+x}}$.

$\lim_{x\to\infty}e^{\frac{2x^2+1}{3x^2+x}}=e^{\lim_{x\to\infty}e^{\frac{2x^2+1}{3x^2+x}}}=e^{2/3}$

[4] **4.** For which real number a is the function

$$f(x) = \begin{cases} x^2 & x \geq 2 \\ \frac{ax+2}{x-1} & x < 2 \end{cases}$$

continuous on $\mathbb{R}$? Justify your answer.

The only place where f may not be continuous is at $x = 2$ where it is required that $\lim_{x\to 2^+} f(x) = \lim_{x\to 2+} x^2 = 4$ is equal to $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{ax+2}{x-1} = 2a+2$. Hence $2a+2=4$ and $a=1$.

5. The price of one share of a certain company increased from \$$e$ on 1 August 2004 to \$$e^3$ on 1 December 2005.

[2] (a) Determine the exponential function which models the growth of this stock price as a function of time elapsed in months since 1 August 2004.

Assume $f(t) = y_0 e^{kt}$ where t is the time in months. Then we are given that $f(0) = e$ and $f(16) = e^3$. Hence, $y_0 = e$ and $ee^{16k} = e^3$ so $16k + 1 = 3$ and $k = 1/8$. So $f(t) = ee^{t/8}$.

[2] (b) Predict the stock price in December 2006.

We wish to know the value $f(28) = ee^{28/8} = e^{9/2}$.

[2] (c) What is the time it takes for the stock to double in price?

This happens when $y_0 e^{kt} = 2y_0$, that is when $e^{kt} = 2$ and hence $kt = \ln 2$ so $t = \frac{\ln 2}{k} = 8 \ln 2$ months.

6. In a proposed taxation scheme, the tax collected on business income of x million dollars is given by the formula

$$T(x) = \begin{cases} 4x - 3 & x < 2 \\ x^2 + 1 & x \geq 2 \end{cases}.$$

[2] (a) Determine the average tax collected if the business income falls within the range of 1 to 3 million dollars.

The average in this range is $\frac{T(3)-T(1)}{3-1} = \frac{9-1}{2} = 4$.

[4] (b) Determine the marginal tax rate if the business income is 2 million dollars. Compute this directly using limits.

We need to compute the limit $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$. We compute each side separately as the function is defined differently on each side.

$$\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{4x - 8}{x - 2} = 4$$
$$\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \frac{x^2 - 4}{x - 2} = 4.$$

Therefore, $\lim_{x\to 2} f(x) = 4$ is the marginal tax rate at $x = 2$.