Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157-3, 1107
Midterm 2
November 3rd, 2010, 11:30 – 12:20

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Last Name (please print): ____________**KEY**_________________

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Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 8 pages with a total of 7 questions. When instructed, please check to make sure your exam is complete.
4. Only the usual writing instruments, this booklet, and an acceptable calculator shall be within your reach.
5. If you run out of space in a problem, use the space on the back of the cover page and clearly indicate where the solution continues.
6. **Only** scientific with no graphing, and programming capabilities are allowed.
7. All other electronic devices must be turned off and out of reach.
8. Speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. **Full marks will be reserved for answers that are correct in all essential details and could be understood by another student without due effort.**
10. You must stop writing when time is called.

Do not write in this table!	
Question	**Marks**
1	**/4**
2	**/4**
3	**/4**
4	**/3**
5	**/5**
6	**/6**
7	**/4**
Total	**/30**

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1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. **[1/2 mark each = 4 marks]**

a) F Suppose *C(x)* is the total cost function for a certain product, then the average cost function is given by $\overline{C}(x) = xC(x)$ where *x* is the number of units sold.

b) T $y = xy^2 + x^3 - 5$ is an implicitly defined relation.

c) F For a differentiable function $y = f(x)$ and the statement $\Delta y \approx f'(x)\Delta x = f'(x)dx = dy$, Δy is referred to as the increment of *y* and measures the actual change in *y* as *x* changes from *x* to Δx,and dy is referred to as the differential of *y* and measures the approximate change in *y* as *x* changes from *x* to Δx.

d) F The Newton-Raphson method states that given a differentiable function *f* the solution to $f(x) = 0$ is approximated by $x_{n+1} = x_n - \dfrac{f'(x_n)}{f(x_n)}$, where x_1 is the initial value, $n \in \mathbb{N}$, and $f'(x_n) \neq 0$.

e) T One of the laws of exponents states that $b^x = e^{\ln b^x} = e^{x \ln b}$, where *b* is the base with $b > 0$ and $b \neq 1$, and $x \in \mathbb{R}$.

f) F One of the laws of logarithms states that $\log_b x = \dfrac{\ln b}{\ln x} \left(= \dfrac{\log_a b}{\log_a x} \right)$, where *a, b* are bases with $a, b > 0$ and $a, b \neq 1$, and $x \in \mathbb{R}$.

g) F $\lim\limits_{n \to \infty} \dfrac{e^n - 1}{n} = 1$.

h) T A logistic growth function is a function that models growth that is proportional to the amount of the quantity present and takes into account other factors that limit the growth.

2. Find the indicated derivative provided it exists. State the domain of the function and the domain of its derivative. Do not simplify the derivative! **[4 marks]**

a) $f(x) = \sqrt[4]{x} \cdot e^{5x}, \; f'(x)$

$$f(x) = \sqrt[4]{x} \cdot e^{5x} = x^{1/4}e^{5x} \text{ with } D_f = \{x \in \mathbb{R} \,/\, x \geq 0\}$$

$$f'(x) = \frac{1}{4}x^{-3/4}e^{5x} + 5x^{1/4}e^{5x} \text{ with } D_{f'} = \{x \in \mathbb{R} \,/\, x > 0\}$$

$$\left[= e^{5x}\left(\frac{1}{4}x^{-3/4} + 5x^{1/4}\right)\right]$$

b) $y = \log_4 (x+5)^3, \; \dfrac{dy}{dx}$

$$y = \log_4 (x+5)^3 \text{ with } D_y = \{x \in \mathbb{R} \,/\, x > -5\}$$

$$\frac{dy}{dx} = \frac{3(x+5)^2}{(x+5)^3 \ln 4} \text{ with } D_{y'} = \{x \in \mathbb{R} \,/\, x > -5\}$$

$$\left[= \frac{3}{(x+5)\ln 4}\right]$$

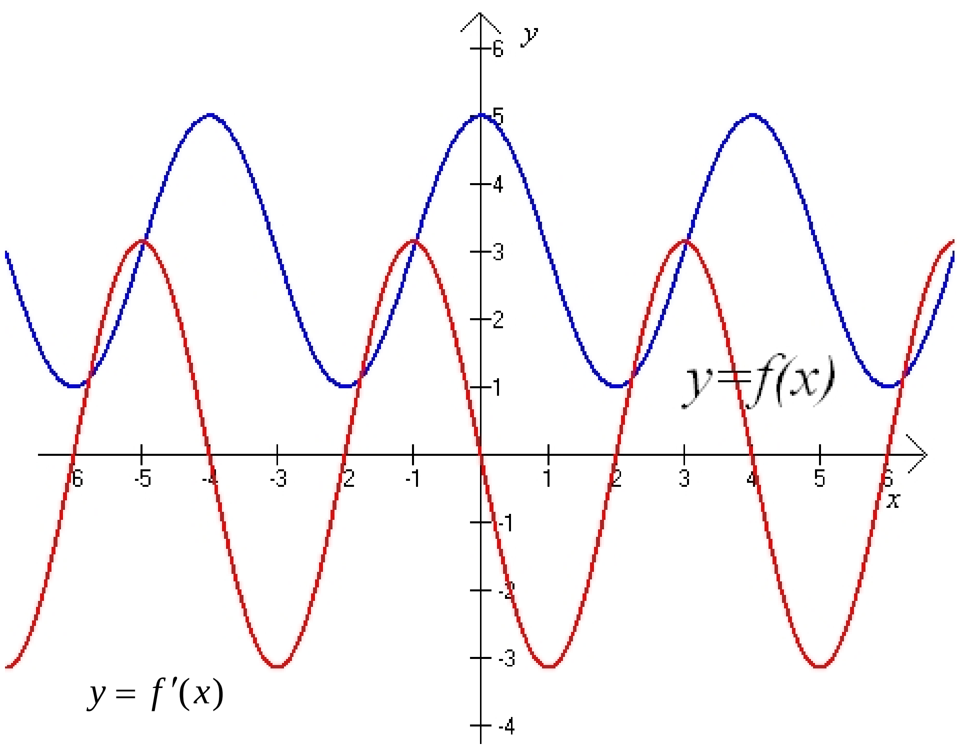
3. The population of Canadians age 55 yr and over as a percent of the total population is approximated by the function $f(t)=10.72(0.9t+10)^{0.3}$ with $0\le t\le 20$, where t is measured in years and $t=0$ corresponding to 2000. Compute both $f'(10)$ and $f''(10)$ accurate to 2 decimal places, and interpret your results. (extended textbook exercise 4.3 #40) **[4 marks]**

$$f(t)=10.72(0.9t+10)^{0.3}$$
$$f'(t)=10.72(0.3)(0.9t+10)^{-0.7}(0.9)=2.8944(0.9t+10)^{-0.7}$$
$$f''(t)=2.8944(-0.7)(0.9t+10)^{-1.7}(0.9)=-1.823472(0.9t+10)^{-1.7}$$

So, $f'(10)=2.8944(0.9(10)+10)^{-0.7}\approx 0.36849$. This says that the rate of change of population with respect to time is increasing by approximately 0.37%/yr.

So, $f''(10)=-1.823472(0.9(10)+10)^{-1.7}\approx -0.01222$. This says that the rate of change of *f'* with respect to time is decreasing by approximately 0.01%/yr^2.

4. Use the graph of $y=f(x)$ below to graph its derivative. (textbook exercise 4.2 #10) **[3 marks]**



5. The demand equation for a certain product is given by $p = 144 - x^2$. Compute the elasticity of demand and determine if whether the demand is elastic, inelastic or unitary at $p = 96$. (textbook exercise 4.1 #28) **[5 marks]**

textbook solution

Solving the demand equation for x, we find

$$x^2 = 144 - p, \text{ or } x = \sqrt{144 - p} \qquad [x \text{ must be nonnegative}]$$

With $x = f(p) = (144 - p)^{1/2}$, we have

$$f'(p) = \frac{1}{2}(144 - p)^{-1/2}(-1) = -\frac{1}{2\sqrt{144 - p}}.$$

Therefore, $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\dfrac{1}{2\sqrt{144-p}}\right)}{\sqrt{144-p}} = \dfrac{p}{2(144-p)}.$

$E(96) = \dfrac{96}{2(48)} = 1$, and so the demand equation is unitary.

instructor solution:

$$\frac{d}{dp}(p) = \frac{d}{dp}\left(144 - x^2\right) \Rightarrow 1 = -2x\frac{dx}{dp} \Rightarrow \frac{dx}{dp} = -\frac{1}{2x}$$

So, $E(x) = -\dfrac{p}{x}\dfrac{dx}{dp} = -\dfrac{144 - x^2}{x}\left(-\dfrac{1}{2x}\right) = \dfrac{144 - x^2}{2x^2}$.

Now, when $p = 96$ then $96 = 144 - x^2 \Rightarrow x = \sqrt{144 - 96} = \sqrt{48}$, since x must be non-negative.

Finally, $E\left(\sqrt{48}\right) = \dfrac{144 - 48}{2 \cdot 48} = 1$, and so the demand equation is unitary.

6. The effective radioactive lifetime of polonium-210 is so short we measure it in days rather than years. The number of radioactive atoms remaining after t days is $y(t)=y_0e^{-5\times10^{-3}t}$. **[1+2+3 marks]**

   a) What is the meaning of y_0?

      y_0 is the initial amount of polonium-210 in atoms.

   b) As a percentage, approximately how much is left of the original amount in 50 days?

      $y(50)=y_0e^{-5\times10^{-3}\times50}\approx0.78y_0$, i.e. about 78% of the original amount y_0 is left after 50 days.

   c) After how many days is 20% of the original amount of polonium-210 left?

$$0.20y_0=y_0e^{-5\times10^{-3}t}$$
$$0.20=e^{-5\times10^{-3}t}$$
$$\ln 0.20=-5\times10^{-3}t$$
$$t=\frac{\ln 0.20}{-5\times10^{-3}}=321.8875825\ldots\approx322 \text{ days}$$

7. Use linear approximation to estimate the value of $f(2.1)$, given that $f(2)=8$ and $f'(x)=\dfrac{3x^2}{\sqrt{x^3+1}}$. **[4 marks]**

$$
\begin{aligned}
f(2.1) &\approx L(2.1) \\
&= f(2)+f'(2)(2.1-2) \\
&= 8+\frac{3\left(2^2\right)}{\sqrt{2^3+1}}(2.1-2) \\
&= 8+4\cdot 0.1 \\
&= 8.4
\end{aligned}
$$