Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157-3, 1107
Midterm 1
October 6^{th}, 2010, 11:30 – 12:20

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Last Name (please print): ____________**KEY**_________________

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Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 8 pages with a total of 7 questions. When instructed, please check to make sure your exam is complete.
4. Only the usual writing instruments, this booklet, and an acceptable calculator shall be within your reach.
5. If you run out of space in a problem, use the space on the back of the cover page and clearly indicate where the solution continues.
6. **Only** scientific with no graphing, and programming capabilities are allowed.
7. All other electronic devices must be turned off and out of reach.
8. Speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. **Full marks will be reserved for answers that are correct in all essential details and could be understood by another student without due effort.**
10. You must stop writing when time is called.

Do not write in this table!

Question	Marks
1	/4
2	/3
3	/3
4	/6
5	/5
6	/5
7	/4
Total	**/30**

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1. Answer **T** (true) or **F** (false) in the boxes provided or leave the box blank. No explanation is necessary. **[1/2 mark each = 4 marks]**

a) $\boxed{\text{T}}$ When $f(-x) = -f(x)$ then f is called an odd function and the graph of $y = f(x)$ is symmetric about the origin.

b) $\boxed{\text{T}}$ $f(x) = |x-1| = \sqrt{(x-1)^2} = \left(\sqrt{x-1}\right)^2$ for all real x in $x \geq 1$.

c) $\boxed{\text{F}}$ The graph of $y = f(x-h)$ is the graph of $y = f(x)$ translated to the left by h units.

d) $\boxed{\text{F}}$ Suppose $\lim\limits_{x\to a} f(x) = L$ and $\lim\limits_{x\to a} g(x) = M$, then $\lim\limits_{x\to a}\left[f(x)/g(x)\right] = \left[\lim\limits_{x\to a} f(x)\right] / \left[\lim\limits_{x\to a} g(x)\right] = L/M$.

e) $\boxed{\text{T}}$ If $\lim\limits_{x\to\infty} f(x) = L$, where L is a real number, then $y = L$ is a horizontal asymptote for the graph of f.

f) $\boxed{\text{F}}$ The derivative (provided it exists) at $x = a$ of a function $y = f(x)$ is the slope of the tangent line to f at any point.

g) $\boxed{\text{F}}$ For differentiable functions f and g, the product rule for differentiation says that $\dfrac{d}{dx}\left[f(x)g(x)\right] = \dfrac{d}{dx}\left[f(x)\right]\dfrac{d}{dx}\left[g(x)\right]$.

h) $\boxed{\text{T}}$ The chain rule in Leibniz notation says that if $a = f(b)$ and $b = g(c)$ then $a = f(g(c))$ and $\dfrac{da}{dc} = \dfrac{da}{db} \cdot \dfrac{db}{dc}$.

2. The supply and demand equations for a certain product are given by $p = -x^2 - 2x + 100$ and $p = 8x + 25$, where x represents the quantity demanded in units of a thousand and p the unit price in dollars. Find the equilibrium quantity and the equilibrium price. (textbook exercise 2.8 #20) **[3 marks]**

We solve the system of equations $p = -x^2 - 2x + 100$ and $p = 8x + 25$.

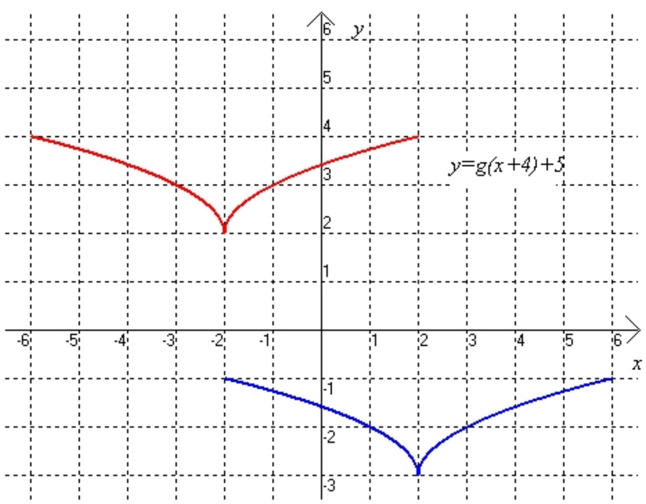
Thus, $-x^2 - 2x + 100 = 8x + 25$, or $\quad x^2 + 10x - 75 = 0$.

Factoring this equation, we have $(x + 15)(x - 5) = 0$, or $x = -15$ and $x = 5$. Rejecting the negative root, we have $x = 5$ and the corresponding value of p is

$$p = 8(5) + 25 = 65.$$

We conclude that the equilibrium quantity is 5000 and the equilibrium price is \$65.

3. Use the graph of $y = g(x)$ below to graph the transformed function $y = g(x+4)+5$. (textbook exercise 2.2 #10) **[3 marks]**



4. Let $f(x)=\begin{cases}\dfrac{x^2-4}{x+2} & \text{if } x\neq -2\\ k & \text{if } x=-2\end{cases}$.

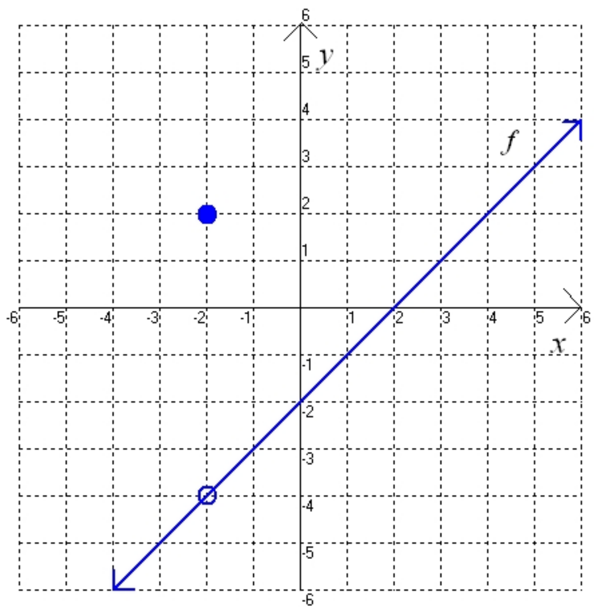
a) For what value of *k* will *f* be continuous on $(-\infty,\infty)$? (textbook exercise 3.2 #78) **[3 marks]**

For $x\neq -2$ we have that $f(x)=\dfrac{x^2-4}{x+2}=\dfrac{(x+2)(x-2)}{x+2}=x-2$ which is a polynomial and so it is continuous for all $x\neq -2$.

For $x=-2$, we need $\lim\limits_{x\to -2} f(x)=f(-2)$. Since $\lim\limits_{x\to -2}\dfrac{x^2-4}{x+2}=\lim\limits_{x\to -2}\dfrac{(x+2)(x-2)}{x+2}=\lim\limits_{x\to -2}(x-2)=-4$ we define $f(-2)=k=-4$ that is $k=-4$. Now $\lim\limits_{x\to -2} f(x)=f(-2)$ and f is continuous at $x=-2$.

Thus we have shown that *f* is continuous for all real numbers *x*.

b) Sketch the labeled graph of *f* in the coordinate system below when $k=2$. **[3 marks]**



5. Find the indicated derivative provided it exists. State the domain of the function and the domain of its derivative. Do not simplify the derivative! **[5 marks]**

a)

$$g(x) = \frac{x-2}{x+3} \text{ with } D_g = \mathbb{R} \setminus \{-3\}$$

$$g'(x) = \frac{(x-2)'(x+3)-(x-2)(x+3)'}{(x+3)^2}$$

$$= \frac{(x+3)-(x-2)}{(x+3)^2}$$

$$= \frac{5}{(x+3)^2} \text{ with } D_{g'} = \mathbb{R} \setminus \{-3\}$$

b)

$$y = \sqrt[5]{3x^2+5x-2} = \left(3x^2+5x-2\right)^{1/5} = \left((3x-1)(x+2)\right)^{1/5} \text{ with } D = \mathbb{R}$$

$$y' = \frac{1}{5}\left(3x^2+5x-2\right)^{-4/5}(6x+5) \text{ with } D = \mathbb{R} \setminus \left\{-2, \frac{1}{3}\right\}$$

6. Describe the following limits in terms of a real number, $\infty, -\infty$, or **DNE** (does not exist). Show your work. **[5 marks]**

a) $\lim_{x\to 1}\frac{3\pi}{2x^3-1}=\frac{3\pi}{2(1)^3-1}=\frac{3\pi}{2-1}=3\pi$

b)

$$\lim_{x\to 3^-}\frac{3-x}{|x-3|}=\lim_{x\to 3^-}\frac{3-x}{-(x-3)}=\lim_{x\to 3^-}1=1$$

$$\lim_{x\to 3^+}\frac{3-x}{|x-3|}=\lim_{x\to 3^+}\frac{3-x}{x-3}=\lim_{x\to 3^+}(-1)=-1$$

Since $\lim_{x\to 3^-}\frac{3-x}{|x-3|}\neq\lim_{x\to 3^+}\frac{3-x}{|x-3|}$ we have that

$\lim_{x\to 3^-}\frac{3-x}{|x-3|}$ DNE.

c)

$$\lim_{x\to\infty}\frac{2x^2-6}{5+x-3x^2}=\lim_{x\to\infty}\frac{x^2\left(2-\frac{6}{x^2}\right)}{x^2\left(\frac{5}{x^2}+\frac{1}{x}-3\right)}$$

$$=\lim_{x\to 4}\frac{2-\cancelto{0}{\frac{6}{x^2}}}{\left(\cancelto{0}{\frac{5}{x^2}}+\cancelto{0}{\frac{1}{x}}-3\right)}$$

$$=\frac{2}{-3}=-\frac{2}{3}$$

7. Use the definition of the derivative to calculate $f'(12)$ for $f(x)=\sqrt{x-8}$.
[4 marks]

$$
\begin{aligned}
f'(12) &= \lim_{x\to 12}\frac{f(x)-f(12)}{x-12}\\
&= \lim_{x\to 12}\frac{\sqrt{x-8}-\sqrt{12-8}}{x-12}\\
&= \lim_{x\to 12}\frac{\sqrt{x-8}-2}{x-12}\\
&= \lim_{x\to 12}\frac{\sqrt{x-8}-2}{x-12}\cdot\frac{\sqrt{x-8}+2}{\sqrt{x-8}+2}\\
&= \lim_{x\to 12}\frac{x-8-4}{(x-12)\left(\sqrt{x-8}+2\right)}\\
&= \lim_{x\to 12}\frac{x-12}{(x-12)\left(\sqrt{x-8}+2\right)}\\
&= \lim_{x\to 12}\frac{1}{\sqrt{x-8}+2}\\
&= \frac{1}{\sqrt{12-8}+2}\\
&= \frac{1}{4}
\end{aligned}
$$

or

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{x+h-8}-\sqrt{x-8}}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{x+h-8}-\sqrt{x-8}}{h}\cdot\frac{\sqrt{x+h-8}+\sqrt{x-8}}{\sqrt{x+h-8}+\sqrt{x-8}}\\
&= \lim_{h\to 0}\frac{(x+h-8)-(x-8)}{h\left(\sqrt{x+h-8}+\sqrt{x-8}\right)}\\
&= \lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h-8}+\sqrt{x-8}\right)}\\
&= \lim_{h\to 0}\frac{1}{\sqrt{x+h-8}+\sqrt{x-8}}\\
&= \frac{1}{\sqrt{x-8}+\sqrt{x-8}}\\
&= \frac{1}{2\sqrt{x-8}}
\end{aligned}
$$

Therefore, $f'(12)=\dfrac{1}{2\sqrt{12-8}}=\dfrac{1}{4}$.