Simon Fraser University, Department of Mathematics, Burnaby Campus

Math 157, Summer 2010

Midterm 1

June 16, 2010, 11:30 a.m.–12:20 p.m., AQ 3182

SOLUTIONS

Last Name (please print): ______________________

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Instructor: Roland Wittler

Instructions

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table.	
Question	**Marks**
1	/6
2	/6
3	/4
4	/4
5	/6
6	/4
Total	**/30**

Question 1: Fill the gaps in the following statements. (6 marks)

(a) Absolute Value: An equation for the x and y coordinates of the circle with centre $C(h, k)$

(1 mark) and radius r is given by: $(\underline{x - h})^2 + (\underline{y - k})^2 = r^2$.

[or $h - x$] [or $k - y$] [or with absolute value bars]

(b) Polynomial Function: Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$ be a poynomial function with $a_n \neq 0$.

(1 mark) (i) Then, $\underline{a_n}$ is called the *leading coefficient*,

and n the <u>degree</u> of p.

(1 mark) (ii) If the sign of the leading coefficient of a polynomial function is positive, its graph goes <u>upward</u> for large x.

(c) One-to-One Function: A function f is one-to-one if $f(x_1) \underline{\neq} f(x_2)$ whenever $x_1 \underline{\neq} x_2$.

(1 mark)

(d) Differentiability and Continuity: $f(x) = |x|$ is <u>covitinuous</u> at $x = 0$ but

(1 mark) not <u>differentiable</u> there.

(e) Composite Function: The domain of $h(x) = (f \circ g)(x) = f(g(x))$ are all x from the

(1 mark) domain of $\underline{g}$ such that $\underline{g(x)}$ is in the domain of f.

Question 2: Marked Equilibrium (6 marks).

The company *Cookagoose* plans to release the brand new *Cook-O-Matic-3000.* The marketing department carried out a telephone survey to estimate the costumers purchase behaviour. They found that the demanded number of Cook-O-Matics would depend on the price p as follows:

$$x(p) = 10\left(\sqrt{1001-p}-1\right),$$

where x denotes the number of Cook-O-Matics measured in units of thousand, and p is the price in dollars.

(a) Determine the demand function $p(x)$ by computing the inverse of $x(p)$.

$$x = 10(\sqrt{1001-p}-1)$$

$$\Leftrightarrow \frac{x}{10} = \sqrt{1001-p}-1$$

$$\Leftrightarrow \frac{x}{10}+1 = \sqrt{1001-p}$$

$$\Leftrightarrow \left(\frac{x}{10}+1\right)^2 = 1001-p$$

$$\Leftrightarrow \left(\frac{x}{10}+1\right)^2 - 1001 = -p$$

$$\Leftrightarrow p(x) = -\left(\frac{x}{10}+1\right)^2 + 1001$$

(3 marks)

(b) Let the supply function be given by $p(x) = 0.5x+922$. Show that $x = 60$ is the equilibrium quantity and give the equilibrium price.

demand: $p(x=60) = -\left(\frac{60}{10}+1\right)^2 + 1001$

$= -7^2 + 1001$

$= 952$

supply: $p(x=60) = 0.5\cdot 60 + 922$

$= 30 + 922$

$= 952$

$(x=60, p=952)$ is the intersection point of demand and supply function. $p = 952$ dollars is the equilibrium price.

(3 marks)

Question 3: Limits (4 marks).

Consider the following function:

$$f(x) = \begin{cases} -x+2 & \text{for } x \le 0 \\ 5 - \dfrac{3}{1+x} & \text{for } x > 0 \end{cases}$$

(a) What is the right-hand limit of f as x approaches 0 from the right?

$$\lim_{x \to 0^+} \left(5 - \frac{3}{1+x}\right) = 5 - \frac{3}{1+0} = 2$$ (0.5 marks)

(b) What is the left-hand limit of f as x approaches 0 from the left?

$$\lim_{x \to 0^-} (-x+2) = 0 + 2 = 2$$ (0.5 marks)

(c) Verify whether f is continuous at $x = 0$.

1. $f(0)$ is defined
2. $\lim_{x\to 0} f(x)$ exists since $\lim_{x\to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ (0.5 marks each)
3. $\lim_{x \to 0} f(x) = 2 = f(0)$
4. $\Rightarrow f$ is continuous at $x = 0$.

(d) Evaluate the limit of f as x approaches positive infinity. (1 mark)

$$\lim_{x\to\infty} f(x) = \lim_{x \to \infty}\left(5 - \frac{3}{1+x}\right) = \lim_{x\to\infty}\left(5 - \frac{3/x}{1/x + 1}\right)$$

$$= 5 - \frac{0}{0+1} = 5$$

Question 4: Parabola (4 marks).

Let $f(x) = \frac{1}{2}(x-4)^2 + 1$. $= \frac{1}{2}(x^2 - 8x + 16) + 1 = \frac{1}{2}x^2 - 4x + 8 + 1 = \frac{1}{2}x^2 - 4x + 9$

(a) Use the four-step process to compute the slope of the tangent line to f at $x = 6$.

1. $f(x+h) = \frac{1}{2}((x+h)-4)^2 + 1 = \frac{1}{2}(x+h)^2 - 4(x+h) + 9 = \frac{1}{2}(x^2 + 2hx + h^2) - 4x + 9 - 4h$
$= \frac{1}{2}x^2 + hx + \frac{1}{2}h^2 - 4x + 9 - 4h$

2. $f(x+h) - f(x) = \cancel{\frac{1}{2}x^2} + hx + \frac{1}{2}h^2 \cancel{- 4x} \cancel{+ 9} - 4h \cancel{- \frac{1}{2}x^2} \cancel{+ 4x} \cancel{- 9} = hx + \frac{1}{2}h^2 - 4h$

3. $\frac{f(x+h) - f(x)}{h} = \frac{hx + 1/2h^2 - 4h}{h} = x + \frac{1}{2}h - 4$

4. $\lim_{h \to 0}\left(x + \frac{1}{2}h - 4\right) = x - 4$

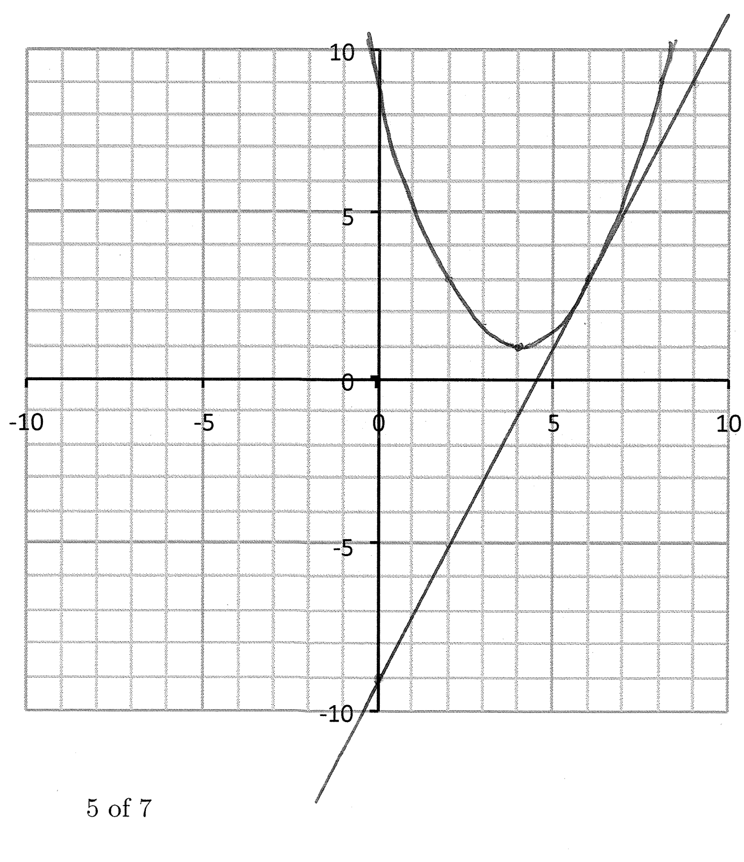
(0.5 marks for each step)

Or shorter: 1. $f(6+h) = \ldots = \frac{1}{2}h^2 + 2h + 3$
2. $f(6+h) - f(6) = \ldots = \frac{1}{2}h^2 + 2h$
3. $\frac{f(6+h) - f(6)}{h} = \frac{1}{2}h + 2$
4. $\lim_{h \to 0}\left(\frac{1}{2}h + 2\right) = 2$

$\Rightarrow f'(6) = 2$

(b) Sketch the graph of f and the tangent line at $x = 6$ on the given coordinate system.

(1 mark for each graph)



Question 5: Derivatives (6 marks).

Compute the derivatives as indicated below. Do not simplify.

(a) $f(x) = (x^2 - 1 + x^3)\sqrt{x^3}$, $f'(x)$

$$= (x^2 - 1 + x^3)x^{3/2}$$

$$f'(x) = (2x + 3x^2)x^{3/2} + (x^2 - 1 + x^3)\frac{3}{2}x^{\frac{1}{2}}$$

$$\left[= (2x + 3x^2)\sqrt{x^3} + (x^2 - 1 + x^3)\frac{3}{2}\sqrt{x} \right]$$

(2 marks)

(b) $s(t) = \dfrac{2t^3}{(t+3)^2}$, $\dfrac{ds}{dt}$

$$\frac{ds}{dt} = \frac{(t+3)^2 \cdot 6t^2 - 2t^3(2t+6)}{(t+3)^4}$$

(2 marks)

(c) $h(x) = \sqrt[3]{3x^3 - 3}$, $h'(x)$

$$h'(x) = \frac{1}{3}(3x^3 - 3)^{-\frac{2}{3}} \cdot 9x^2 \quad \left[= \frac{9x^2}{3\sqrt[3]{(3x^3-3)^2}} \right]$$

(2 marks)

Question 6: Selling pizza (4 marks).

Toni sells pizza. He estimates the fixed cost for electricity, rent etc. to be 200 dollars per day and the cost of material for a pizza to be 2 dollars.

(a) Give the cost function $C(x)$ for producing x pizzas a day.

$$C(x) = 200 + 2x$$

(1 mark)

(b) What is the average cost $\overline{C}$ for producing 100 pizzas a day?

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{200+2x}{x} = \frac{200}{x} + 2$$

$$\overline{C}(100) = \frac{200}{100} + 2 = 4$$

(1 mark)

(c) What is the marginal cost C' for producing the 100th pizza on a day?

$$C'(x) = 2$$

$$C'(100) = 2$$

(1 mark)

(d) Independently of the demand and the market equilibrium, Toni sells his pizza for 5 dollars each. Give the revenue function $R(x)$ and the resulting profit function $P(x)$.

$$R(x) = 5x$$

$$P(x) = R(x) - C(x) = 5x - (200 + 2x) = 3x - 200$$

(1 mark)