Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157 - D100 Spring09 Calculus I for the Social Sciences
Midterm 1 – Version 1
February 4th 2009, 11:30–12:20

Last Name (please print):	Solutions version 1
First Name (please print):	______________________
Student number:	______________________
Signature: **(do not sign before your ID is checked)**	______________________
Instructor:	Y. van Gennip

Instructions:

1. **Do not open this booklet until told to do so.**
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. **Show all your work!**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no graphing, differentiation, and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet, and an acceptable calculator shall be within reach of a student during the examination.
8. During the examination speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your best!

Do not write in this table!	
Question	**Marks**
1	**/3**
2	**/6**
3	**/5**
4	**/6**
5	**/5**
6	**/6**
Total	**/31**

1. Answer the following questions with "true" or "false". No explanation is necessary. **[1/2 mark each = 3 marks]**

   (a) Two lines are perpendicular if and only if the product of their slopes is -1, or if one of the lines is horizontal and the other vertical.

   True

   (b) Let f be any function and h a positive constant. The graph of $y = f(x-h)$ is the graph of $y = f(x)$ translated to the left by an amount h.

   False, the graph is translated to the right.

   (c) The domain of the function $f(x) = \sqrt{8-6x}$ is the set $\left\{x < \frac{4}{3}\right\}$.

   False, the domain is $\left\{x \leq \frac{4}{3}\right\}$.

   (d) $\cos\frac{43\pi}{6} = -\sqrt{3/4}$

   True: $\cos\frac{43\pi}{6} = \cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{1}{2}\sqrt{3} = -\sqrt{3/4}$

   (e) If $h(x) = e^{2x}$, then $h'(\ln 3) = \lim\limits_{b \to \ln 3} \frac{e^{2b}-9}{b-\ln 3}$.

   True:

$$\begin{aligned} h'(\ln 3) &= \lim_{b \to \ln 3} \frac{h(b) - h(\ln 3)}{b - \ln 3} = \lim_{b \to \ln 3} \frac{e^{2b} - e^{2\ln 3}}{b - \ln 3} \\ &= \lim_{b \to \ln 3} \frac{e^{2b} - \left(e^{\ln 3}\right)^2}{b - \ln 3} = \lim_{b \to \ln 3} \frac{e^{2b} - 3^2}{b - \ln 3} \\ &= \lim_{b \to \ln 3} \frac{e^{2b} - 9}{b - \ln 3}. \end{aligned}$$

   (f) If $f'(x)$ exists then the function f is continuous at the point x.

   True

2. Draw the graphs of the following functions.

   Indicate pertinent details (for example x- and y-intercepts and horizontal and vertical asymptotes, if there are any). **[3 marks each = 6 marks]**

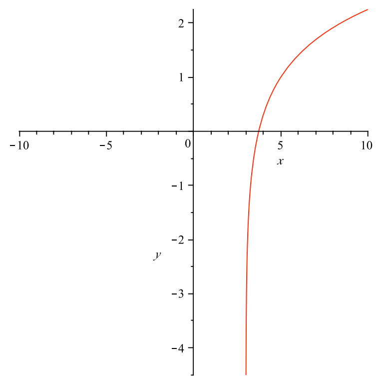
   (a) $f(x) = \ln(\frac{x-3}{2}) + 1$

   x-intercept:

   $f(x) = \ln\left(\dfrac{x-3}{2}\right)+1 = 0 \Leftrightarrow \ln\left(\dfrac{x-3}{2}\right) = -1 \Leftrightarrow \dfrac{x-3}{2} = e^{-1} \Leftrightarrow x = 2e^{-1}+3$

   The domain of the function f is $\left\{\frac{x-3}{2} > 0\right\} = \{x > 3\}$ and so there is no y-intercept.

   There is a vertical asymptote when $\frac{x-3}{2} = 0$, thus at $x = 3$.



   (b) $g(x) = \frac{x^2-2x-8}{x-4}$

   First note that if $x \neq 4$, then

$$g(x) = \frac{x^2-2x-8}{x-4} = \frac{(x-4)(x+2)}{x-4} = x+2.$$

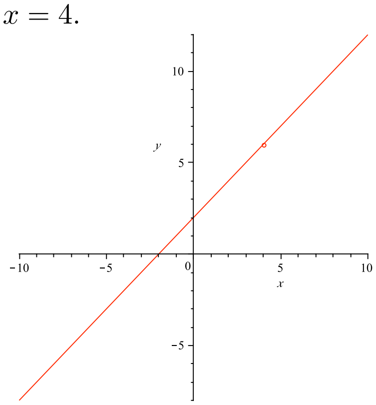
   x-intercept:

$$g(x) = 0 \Leftrightarrow x+2 = 0 \text{ and } x \neq 4 \Leftrightarrow x = -2.$$

   y-intercept:

$$g(0) = 0+2 = 2.$$

   The domain of the function g is $\{x \neq 4\}$, so there is a hole at $x = 4$.

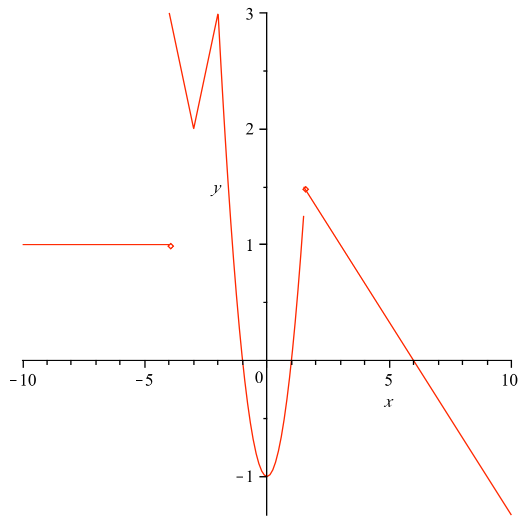


3. Let

$$f(x) = \begin{cases} 1 & \text{if } x \le -4, \\ 2 + |x+3| & \text{if } -4 < x \le -2, \\ x^2 - 1 & \text{if } -2 < x < \frac{3}{2}, \\ -\frac{1}{3}x + 2 & \text{if } x \ge \frac{3}{2}. \end{cases}$$

[5 marks]

(a) Graph the function f in the coordinate system given below.



(The little diamonds indicate the value of f at $x = -4$ and $x = \frac{3}{2}$)

(b) At which points is f discontinuous, if any? The constant function

$y = 1$ is continuous on the open interval $(-\infty, 4)$, the absolute value function $y = 2 + |x+3|$ is continuous on the open interval $(-4, -2)$, the quadratic function $y = x^2 - 1$ is continuous on the open interval $(-2, \frac{3}{2})$ and the linear function $y = -\frac{1}{3}x + 2$ is continuous on the open interval $(\frac{3}{2}, \infty)$. Therefore we only need to consider the points $x = -4$, $x = -2$, and $x = \frac{3}{2}$ as possible points of discontinuity.

We start with the point $x = -4$. The function f is defined at $x = -4$: $f(-4) = 1$. Next we check whether the limit exists. $\lim\limits_{x \to -4^-} f(x) = \lim\limits_{x \to -4^-} 1 = 1$ and $\lim\limits_{x \to -4^+} f(x) = \lim\limits_{x \to -4^+} 2 + |x+3| = 3$. Therefore the limit $\lim\limits_{x \to -4} f(x)$ does not exist and the function is discontinuous at $x = -4$.

Now we check the point $x = -2$. The function f is defined at this point: $f(-2) = 2 + |-2+3| = 2 + 1 = 3$. $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} 2 + |x+3| = 3$ and $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} x^2 - 1 = 3$ and so $\lim\limits_{x \to -2} f(x) = 3 = f(3)$. The function is continuous at $x = -2$.

Finally we check the point $x = \frac{3}{2}$. The function is defined at this point: $f(\frac{3}{2}) = -\frac{1}{3}\frac{3}{2} + 2 = \frac{3}{2}$. $\lim\limits_{x \to \frac{3}{2}^-} f(x) = \lim\limits_{x \to \frac{3}{2}^-} x^2 - 1 = \frac{5}{4}$ and

$\lim_{x \to \frac{3}{2}^+} f(x) = \lim_{x \to \frac{3}{2}^+} -\frac{1}{3}x + 2 = \frac{3}{2}$ and so the limit $\lim_{x \to \frac{3}{2}} f(x)$ does not exist and the function is discontinuous at $x = \frac{3}{2}$.

We conclude that f is discontinuous at $x = -4$ and $x = \frac{3}{2}$.

(c) Does $\lim_{x \to -4} f(x)$ exist? If so, compute it. If not, explain why not.

Since $\lim_{x \to -4^-} f(x) = 1 \neq 3 = \lim_{x \to -4^+} f(x)$, the limit does not exist.

4. A fruit shop sells strawberries all year through. Because strawberries are more expensive to produce during the colder months of the year the shop owner varies the price p in dollars for which he sells 100 gram of strawberries according to the formula

$$p(t) = 5 + 2\cos\left(\frac{\pi}{6}t\right),$$

where t denotes the number of months counted from January 1st 2007. **[6 marks]**

(a) What is the period of p and what is the amplitude?

The period of p is 12 and the amplitude is 2.

(b) To what date corresponds $t = 4$?

$t = 4$ corresponds to four months after January 1st 2007 which is May 1st 2007.

(c) What will 1 kilogram of strawberries cost on July 1st 2009?

July 1st 2009 corresponds to $t = 30$. $p(30) = p(6) = 3$, so 100 grams of strawberries will cost 3 dollars. Therefore 1 kilogram of strawberries will cost 30 dollars on July 1st 2009.

(d) On October 1st 2008 a customer came to the shop complaining that the strawberries were twice as expensive as three months earlier. Was he correct?

October 1st 2008 corresponds to $t = 21$. Three months earlier, July 1st 2008, corresponds to $t = 18$. We compute $p(18) = p(6) = 3$ and $p(21) = p(9) = 5$. The customer was not correct.

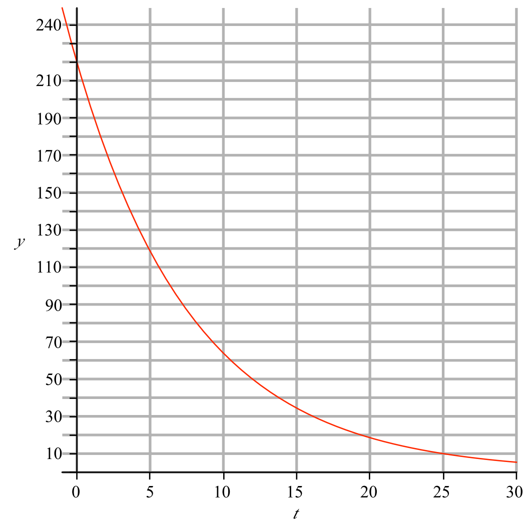
(e) What is the average rate of change for the price of 100 grams of strawberries between October 1st 2007 and March 1st 2009?

October 1st 2007 corresponds to $t = 9$ and March 1st 2009 corresponds to $t = 26$. We compute

$$\frac{p(26) - p(9)}{26 - 9} = \frac{p(2) - p(9)}{17} = \frac{6 - 5}{17} = \frac{1}{17}$$

and thus the average rate of change in the price of 100 grams of strawberries is $\frac{1}{17}$ dollars per month.

5. Ann has baked a pie in her oven. Given is the graph of $y = T(t)$, where $T(t)$ is the temperature of the pie in degrees Celsius, t minutes after taking it out of the oven. [**5 marks**]



(a) How hot is the pie when Ann takes it out of the oven?

We read from the graph that $T(0) = 220$, so the temperature of the pie is 220 degrees Celsius.

(b) How long does it take for the pie to cool down to 10 degrees Celsius?

We read from the graph that $T(25) = 10$, so it takes 25 minutes for the pie to cool down to 10 degrees Celsius.

(c) Assume that the temperature decreases exponentially. Write down a formula for $T(t)$.

The temperature decreases exponentially, so the formula is of the form $T(t) = T_0 e^{-kt}$. $T_0 = T(0) = 220$ as we know from question (a). We use the answer from question (b) to compute

$$10 = T(25) = 220e^{-25k} \Leftrightarrow \frac{1}{22} = e^{-25k} \Leftrightarrow -\ln 22 = \ln \frac{1}{22} = -25k$$
$$\Leftrightarrow k = \frac{1}{25} \ln 22.$$

Therefore the formula for the temperature is given by

$$T(t) = 220e^{-\frac{t}{25} \ln 22}.$$

(d) Ann wants to serve the pie when it is cooled down to 35 degrees Celsius. Use your formula for $T(t)$ to calculate how long she has to wait after taking it out of the oven. Give your answer in minutes accurate up to two decimals.

$$35 = T(t) = 220e^{-\frac{t}{25}\ln 22} \Leftrightarrow \frac{7}{44} = e^{-\frac{t}{25}\ln 22} \Leftrightarrow \ln\frac{7}{44} = -\frac{t}{25}\ln 22$$
$$\Leftrightarrow t = 25\frac{-\ln\frac{7}{44}}{\ln 22} = 25\frac{\ln\frac{44}{7}}{\ln 22} \approx 14.87.$$

So Ann has to wait approximately 14.87 minutes.

6. Compute the following limits. **[2 marks each = 6 marks]**

(a) $\lim\limits_{x\to 1} \frac{x-5\sqrt{x}+4}{1-\sqrt{x}} =$

$$\lim_{x\to 1} \frac{x-5\sqrt{x}+4}{1-\sqrt{x}} = \lim_{x\to 1} \frac{(1-\sqrt{x})(4-\sqrt{x})}{1-\sqrt{x}} = \lim_{x\to 1} (4-\sqrt{x}) = 3$$

(b) $\lim\limits_{x\to 3\pi} \log_{13}\left(\sec^4(x)\right) =$

$$\begin{aligned}
\lim_{x\to 3\pi} \cos(x) &= \cos(3\pi) = \cos(\pi) = -1,\\
\lim_{x\to 3\pi} \sec(x) &= \lim_{x\to 3\pi} \frac{1}{\cos(x)} = \frac{1}{\lim\limits_{x\to 3\pi} \cos(x)} = \frac{1}{-1} = -1,\\
\lim_{x\to 3\pi} \sec^4(x) &= \left(\lim_{x\to 3\pi} \sec(x)\right)^4 = (-1)^4 = 1,\\
\lim_{x\to 3\pi} \log_{13}\left(\sec^4(x)\right) &= \log_{13}\left(\lim_{x\to 3\pi} \sec^4(x)\right) = \log_{13} 1 = 0.
\end{aligned}$$

(c) $\lim\limits_{x\to\infty} e^{-\frac{1}{x}} =$

$$\begin{aligned}
\lim_{x\to\infty} \left(-\frac{1}{x}\right) &= 0,\\
\lim_{x\to\infty} e^{-\frac{1}{x}} &= e^{\lim\limits_{x\to\infty}\left(-\frac{1}{x}\right)} = e^0 = 1.
\end{aligned}$$