

**MATH 157 - D100 Spring09 Calculus I for the Social Sciences**

Midterm 1 – Version 1

February 4th 2009, 11:30–12:20

Last Name (please print): \_\_\_\_\_

First Name (please print): \_\_\_\_\_

Student number: \_\_\_\_\_

Signature: \_\_\_\_\_

**(do not sign before your ID is checked)**

Instructor:

Y. van Gennip

**Instructions:**

1. **Do not open this booklet until told to do so.**

2. Fill in the above box.

3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.

4. **Show all your work!**

5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.

6. **Only** scientific, non-programmable calculators with no graphing, differentiation, and integration capabilities are allowed.

7. No book, paper, or device, other than the usual writing instruments, this booklet, and an acceptable calculator shall be within reach of a student during the examination.

8. During the examination speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.

9. Try your best!

Do not write in this table!	
Question	Marks
1	/3
2	/6
3	/5
4	/6
5	/5
6	/6
<b>Total</b>	<b>/31</b>

1. Answer the following questions with "true" or "false". No explanation is necessary. **[1/2 mark each = 3 marks]**
- (a) Two lines are perpendicular if and only if the product of their slopes is -1, or if one of the lines is horizontal and the other vertical.
- (b) Let  $f$  be any function and  $h$  a positive constant. The graph of  $y = f(x - h)$  is the graph of  $y = f(x)$  translated to the left by an amount  $h$ .
- (c) The domain of the function  $f(x) = \sqrt{8 - 6x}$  is the set  $\{x < \frac{4}{3}\}$ .
- (d)  $\cos \frac{43\pi}{6} = -\sqrt{3/4}$
- (e) If  $h(x) = e^{2x}$ , then  $h'(\ln 3) = \lim_{b \rightarrow \ln 3} \frac{e^{2b} - 9}{b - \ln 3}$ .
- (f) If  $f'(x)$  exists then the function  $f$  is continuous at the point  $x$ .

2. Draw the graphs of the following functions. Indicate pertinent details (for example  $x$ - and  $y$ -intercepts and horizontal and vertical asymptotes, if there are any). [**3 marks each = 6 marks**]

(a)  $f(x) = \ln\left(\frac{x-3}{2}\right) + 1$

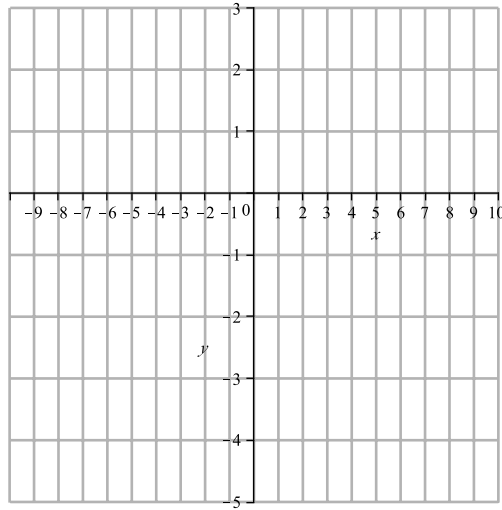
(b)  $g(x) = \frac{x^2-2x-8}{x-4}$

3. Let

$$f(x) = \begin{cases} 1 & \text{if } x \leq -4, \\ 2 + |x + 3| & \text{if } -4 < x \leq -2, \\ x^2 - 1 & \text{if } -2 < x < \frac{3}{2}, \\ -\frac{1}{3}x + 2 & \text{if } x \geq \frac{3}{2}. \end{cases}$$

[5 marks]

(a) Graph the function  $f$  in the coordinate system given below.



(b) At which points is  $f$  discontinuous, if any?

(c) Does  $\lim_{x \rightarrow -4} f(x)$  exist? If so, compute it. If not, explain why not.

4. A fruit shop sells strawberries all year through. Because strawberries are more expensive to produce during the colder months of the year the shop owner varies the price  $p$  in dollars for which he sells 100 gram of strawberries according to the formula

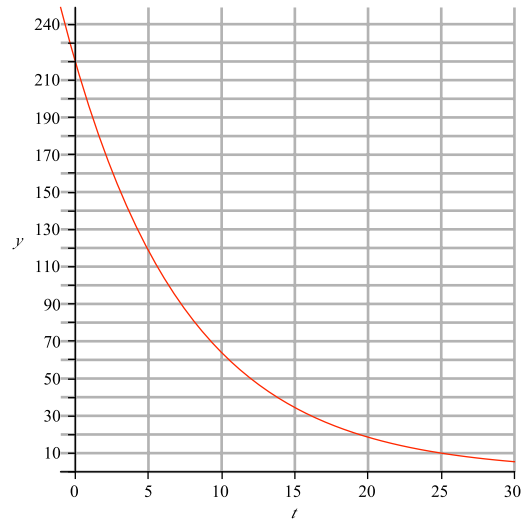
$$p(t) = 5 + 2 \cos \left( \frac{\pi}{6} t \right),$$

where  $t$  denotes the number of months counted from January 1st 2007.

**[6 marks]**

- (a) What is the period of  $p$  and what is the amplitude?
  
  
  
  
  
  
  
  
  
  
- (b) To what date corresponds  $t = 4$ ?
  
  
  
  
  
  
  
  
  
  
- (c) What will 1 kilogram of strawberries cost on July 1st 2009?
  
  
  
  
  
  
  
  
  
  
- (d) On October 1st 2008 a customer came to the shop complaining that the strawberries were twice as expensive as three months earlier. Was he correct?
  
  
  
  
  
  
  
  
  
  
- (e) What is the average rate of change for the price of 100 grams of strawberries between October 1st 2007 and March 1st 2009?

5. Ann has baked a pie in her oven. Given is the graph of  $y = T(t)$ , where  $T(t)$  is the temperature of the pie in degrees Celsius,  $t$  minutes after taking it out of the oven. [5 marks]



- (a) How hot is the pie when Ann takes it out of the oven?
- (b) How long does it take for the pie to cool down to 10 degrees Celsius?
- (c) Assume that the temperature decreases exponentially. Write down a formula for  $T(t)$ .
- (d) Ann wants to serve the pie when it is cooled down to 35 degrees Celsius. Use your formula for  $T(t)$  to calculate how long she has to wait after taking it out of the oven. Give your answer in minutes accurate up to two decimals.

6. Compute the following limits. [**2 marks each = 6 marks**]

(a)  $\lim_{x \rightarrow 1} \frac{x - 5\sqrt{x} + 4}{1 - \sqrt{x}} =$

(b)  $\lim_{x \rightarrow 3\pi} \log_{13}(\sec^4(x)) =$

(c)  $\lim_{x \rightarrow \infty} e^{-\frac{1}{x}} =$