Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157 - D100 Spring09 Calculus I for the Social Sciences

Final Exam

April 11th 2009, 8:30–11:30

Last Name (please print):	____________________
First Name (please print):	____________________
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Student number:	____________________
Signature: **(do not sign before your ID is checked)**	____________________
Instructor:	Y. van Gennip

Instructions:

1. **Do not open this booklet until told to do so.**
2. Fill in the above box. Please use the name under which you are registered.
3. This exam contains 16 pages with a total of 12 questions. Once the exam begins please check to make sure your exam is complete.
4. **Show all your work! Justify your answer unless it is specifically stated that you do not need to.**
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no graphing, differentiation, and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet, and an acceptable calculator shall be within reach of a student during the examination.
8. During the examination speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your best!

Do not write in this table!						
Question	1	2	3	4	5	6
Marks	**/16**	**/4**	**/8**	**/9**	**/6**	**/6**
Question	7	8	9	10	11	12
Marks	**/15**	**/8**	**/7**	**/9**	**/6**	**/6**
Total	**/100**					

1. Determine if the following statements are true or false. No explanation is necessary. **1 mark each = 16 marks**

   (a) Newton's method is a method to compute limits.

   (b) If $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$, then the function f is continuous at $x = c$.

   (c) If demand for a product is inelastic, then an increase in the price of the product leads to an increase in the total revenue.

   (d) If a function is concave up on an open interval, then the function is increasing on that interval.

   (e) If $f(c)$ is a relative minimum of the function f, then $f(x) \geq f(c)$ for all x in the domain of f.

   (f) A function that is continuous on a closed interval has an absolute maximum and an absolute minimum on that interval.

   (g) The derivative of a function f at x is given by

   $$f'(x) = \lim_{h \to 0} \frac{f(x) - f(x+h)}{h}.$$

   (h) The present value of an annuity of payments of R dollars each, made at the end of each period for n consecutive interest periods at a rate of interest i per period is $P = R\left(\frac{1-(1+i)^{-n}}{i}\right)$ dollars.

Question 1 continues on the following page.

(i) The economic lot size is the is the number of products that should be produced in each batch to minimize the total cost of producing and storing the products.

(j) The graph of the function $f(x) = \sin x$ is concave up if and only if $f(x) < 0$.

(k) A function is a rule that assigns to each element from one set at least one element from another set.

(l) For $a > 0$, $a \neq 1$, and $x > 0$ the expression $y = \log_a x$ means $a^y = x$.

(m) Let f be a function with domain $D_f = \mathbb{R} \setminus \{5\}$ then f has a vertical asymptote at $x = 5$.

(n) For a linear function the instantaneous rate of change is the same at each number in its domain.

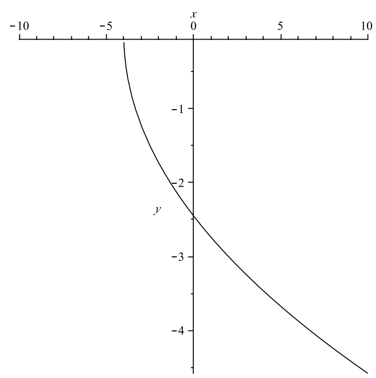
(o) Let f be a function with domain $D_f = \mathbb{R}$. If $f'(x) < 0$ for all $x < 4$, $f'(x) > 0$ for all $x > 4$, and $f(4) = 600,000$, then $600,000$ is the absolute minimum for f.

(p) If P dollars is invested at a yearly rate of interest r per year, compounded m times per year for t years, the compound amount is $A = P\left(1 + \frac{r}{m}\right)^{tm}$ dollars.

2. Match each of the graphs below with one of the following functions.

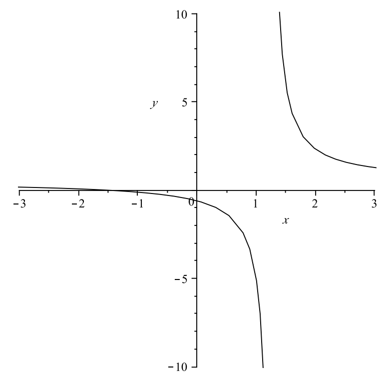
$$
\begin{aligned}
f(x) &= x^9 - \frac{15}{8}x^7 + \frac{273}{256}x^5 - \frac{205}{1024}x^3 + \frac{9}{1024}x,\\
g(x) &= -\sqrt{\frac{3}{2}x + 6},\\
h(x) &= x^7 - \frac{67}{61}x^6 - 9x^5 + x^4 + x^3 + \frac{691}{1313}x,\\
k(x) &= \frac{6}{5}\tan\left(\frac{x-3}{2}\right),\\
l(x) &= \frac{2x+3}{4x-5}.
\end{aligned}
$$

Note that there is one more function given than graphs. No explanation is necessary. **[1 mark each = 4 marks]**



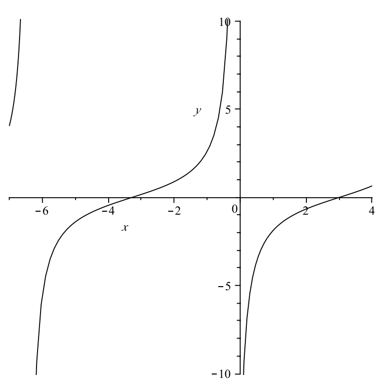
(a)

$y = \ldots$



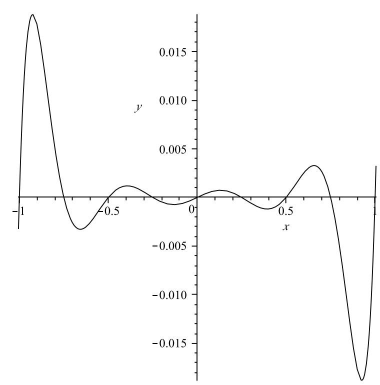
(b)

$y = \ldots$



(c)

$y = \ldots$



(d)

$y = \ldots$

3. Compute the following limits. **2 marks each = 8 marks**

(a) $\lim\limits_{x\to 2} \frac{x^2+2x-8}{x-2}$.

(b) $\lim\limits_{x\to \frac{3}{2}\pi} e^{\frac{\cos x}{x}}$.

(c) $\lim\limits_{x\to -\infty} \frac{8x^8-45x^6+3.3x^2-123}{9x^8+x^7-654x^6+9.81x^4-x^2}$

(d) $\lim\limits_{x\to 2} f(x)$, where

$$f(x) = \begin{cases} 3e^{\ln(\frac{x}{3})} & \text{if } 0 < x \leq 2, \\ x^2+7x-16 & \text{if } x > 2. \end{cases}$$

4. Find the indicated derivatives. You do not need to simplify. **3 marks each = 9 marks**

(a) $f(x) = 73.4\sqrt[5]{x^3} - x\cos x$. Find $f'(x)$.

(b) $g(x) = \frac{2^x}{x^2+x+1}$. Find $g'(x)$.

(c) $h(x) = (x-1)\ln\left(x^2\right)$. Find $h'(x)$.

5. Find the indicated derivatives. You do not need to simplify. **3 marks each = 6 marks**

   (a) $yx = x^2 + 3y^2x^3$. Find $\frac{dy}{dx}$.

   (b) $f(x) = 3\sin x$. If n is a positive and even integer, find $f^{(n)}(x)$.

6. Given is the function

$$f(x) = \sqrt{2x + 8}.$$

6 marks

(a) Give the equation of the tangent line to the graph of f at $x = 12$.

(b) Give the equation of the tangent line to the graph of f at $x = -4$.

7. Given is the function

$$f(x) = \frac{4(4+3x)}{(x+2)^2}.$$

The first and second derivative are given by

$$f'(x) = \frac{-4(3x+2)}{(x+2)^3}, \qquad f''(x) = \frac{24x}{(x+2)^4}.$$

[15 marks]

(a) What is the domain of f?

(b) Compute any x- and y-intercepts of f.

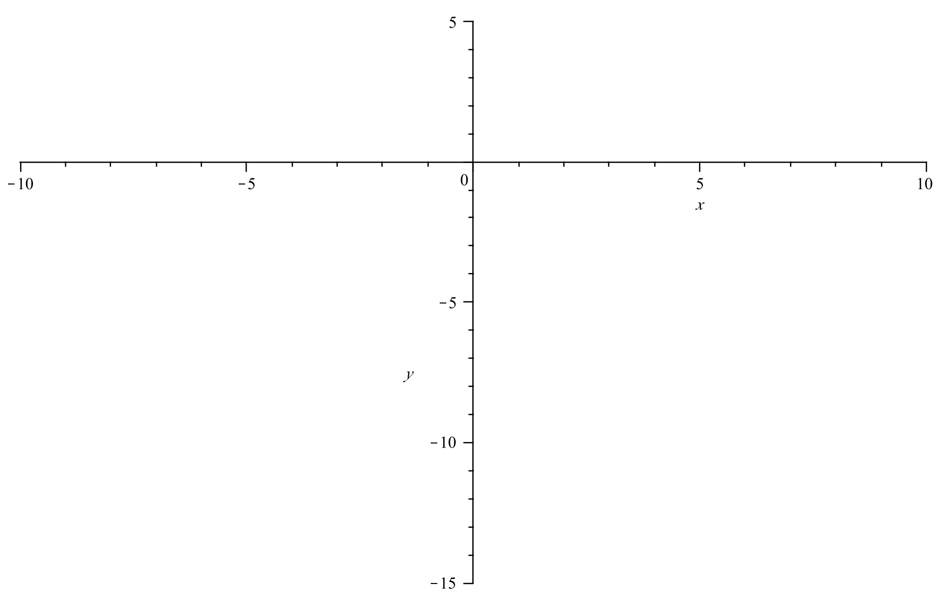
(c) Find all horizontal and vertical asymptotes of f.

Question 7 continues on the following page.

(d) Find the intervals on which f is increasing and decreasing. Find any relative extrema of f and identify them as maxima or minima.

(e) Find all intervals where f is concave up or concave down and all inflection points.

(f) Use the information you have found to sketch the graph of f in the coordinate system below.



8. A pencil producing company has researched the demand of its customers and found that the demand q, given in units of 100,000 pencils per year, depends on the price p per pencil in dollars in the following way:

$$q = f(p) = 24p - 16p^2, \qquad p \in \left(0, \frac{3}{2}\right).$$

8 marks

(a) Give an expression $E(p)$ for the elasticity of demand in terms of the price p and determine for which price intervals the demand is elastic and for which it is inelastic.

(b) Determine the price at which the demand is at its maximum and the maximum value of the demand.

(c) The company can satisfy every demand, i.e. it sells exactly the number of pencils that is demanded, no matter how high the demand is. For which price should the pencils be sold so that the revenue is at a maximum? What is the revenue in that case?

9. Given is the function

$$f(x) = \frac{1}{3}x^3 + 1.75x^2 - 15x + 1$$

and its derivative

$$f'(x) = x^2 + 3.5x - 15.$$

The function f has a root between $x = 2$ and $x = 5$. We are going to use Newton's method to approximate this root. **7 marks**

(a) We start by making an initial guess. Explain which of the following three choices is the best initial guess: $c_1 = -6$, $c_1 = 2.5$, or $c_1 = 4.2$.

(b) Starting from the initial guess you chose in 9a compute three further approximations c_2, c_3, c_4 of the root, using Newton's method.

(c) Based on your answers in 9b give an approximation of the root we are looking for, with two decimals accuracy.

10. Jack wants to be able to buy a new car for $\$50,000$ in five years time. If he invests today then he will get a yearly interest rate of 3%, which is compounded every half year. **9 marks**

   (a) Give an expression for $A(P)$, the compound amount in dollars after five years, if Jack invests P dollars today. Hint: Have a look back at question 1 of this exam.

   (b) How much does Jack need to invest today to have $\$50,000$ in five years time? Give your answer accurate to the nearest cent.

   (c) Another option Jack considers, is to set up an annuity. He computes that if he pays R dollars at the end of each year, with the first payment being a year from now, then the amount of the annuity after five years as a function of R is $S = R\left(\frac{(1.03)^5-1}{0.03}\right)$ dollars. What amount does Jack need to pay per year for the annuity to have a value of $\$50,000$ after five years? Give your answer accurate to the nearest cent.

Question 10 continues on the following page.

(d) What is the present value of the annuity if Jack yearly pays the amount you found in 10c? Give your answer accurate to the nearest cent. Hint: Have another look at question 1 of this exam.

(e) Explain which is the cheaper choice for Jack, an investment today or yearly payments for the annuity?

11. Given is the function

$$f(x) = \sin(12\pi x).$$

6 marks

(a) Use differentials (i.e. linear approximation) to approximate the value of $f\left(\frac{3}{4} + \Delta x\right)$, for the following values of Δx:

i. $\Delta x = 0.1$

ii. $\Delta x = 0.4$

iii. $\Delta x = 0.05$

Give your answer accurate up to three decimals.

(b) Explain for which of the cases in 11a the linear approximation via differentials is not a good way to approximate the value of $f(\frac{3}{4} + \Delta x)$.

12. A manufacturer of handcrafted wine racks has determined that the cost in dollars to produce x units per month is given by

$$C = 0.3x^2 + 11,000.$$

6 marks

(a) How fast is the cost per month changing when production is increasing at a rate of 10 units per month and the production level is 75 units?

(b) The manufacturer has also found a relation between the cost C and revenue R (both in dollars) in one month:

$$C = \frac{R^2}{200,000} + 17500.$$

Find the rate of change of revenue with respect to time when the monthly revenue is $\$25,000$ and the cost is changing at the rate you found in question 12a.