Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157-3, Summer 2008
Midterm 2
July 2nd, 2008, 11:30 – 12:20

Last Name (please print): ______________________

First Name (please print): ______________________

Student Number: ______________________

Instructor: P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 5 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. Only scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table!	
Question	Marks
1	/7
2	+ /12
3	/4
4	/7
5	/10
Total	/40

1. Suppose the demand function for a certain product is given by $p = 4000 - 5x^2$. This equation tells us the price that must be charged per unit to sustain a demand of x units. **[marks]**

a) Write down the formula for the elasticity of demand E.

$$E = -\frac{p}{x}\frac{dx}{dp}$$

b) Find the elasticity of demand E as a function of x.

$$p = 4000 - 5x^2$$

$$\frac{dp}{dp} = \frac{d}{dp}(4000 - 5x^2)$$

$$1 = -10x \frac{dx}{dp}$$

$$\Rightarrow \frac{dx}{dp} = -\frac{1}{10x}$$

$$E = -\frac{p}{x}\frac{dx}{dp}$$

$$E = -\frac{4000 - 5x^2}{x} \cdot \left(-\frac{1}{10x}\right)$$

$$E = \frac{4000 - 5x^2}{10x^2}$$

c) Find and interpret the elasticity of demand when $x = 20$.

$$E(20) = \frac{4000 - 5(20)^2}{10 \cdot 20^2} = \frac{2000}{4000} = \frac{1}{2} < 1$$

This means that the demand is inelastic.

(i.e. the percentage change in demand is less than the percentage change in price OR the total revenue increases as price increases)

2. Find the indicated derivative provided it exists. Do not simplify. **[12 marks]**

a) $f(x)=xe^{4x-3}$, $f'(x)$

$f'(x) = e^{4x-3} + xe^{4x-3}(4)$

b) $y=\log_3\dfrac{(x+2)^5}{x-4}$, y'

$y = 5\log_3(x+2) - \log_3(x-4)$

$y' = \dfrac{5}{(\ln 3)(x+2)} - \dfrac{1}{(\ln 3)(x-4)}$

c) $g(x)=\tan x$, $g''(x)$

$g'(x) = \sec^2 x$

$g''(x) = 2\sec x \cdot \sec x \tan x$

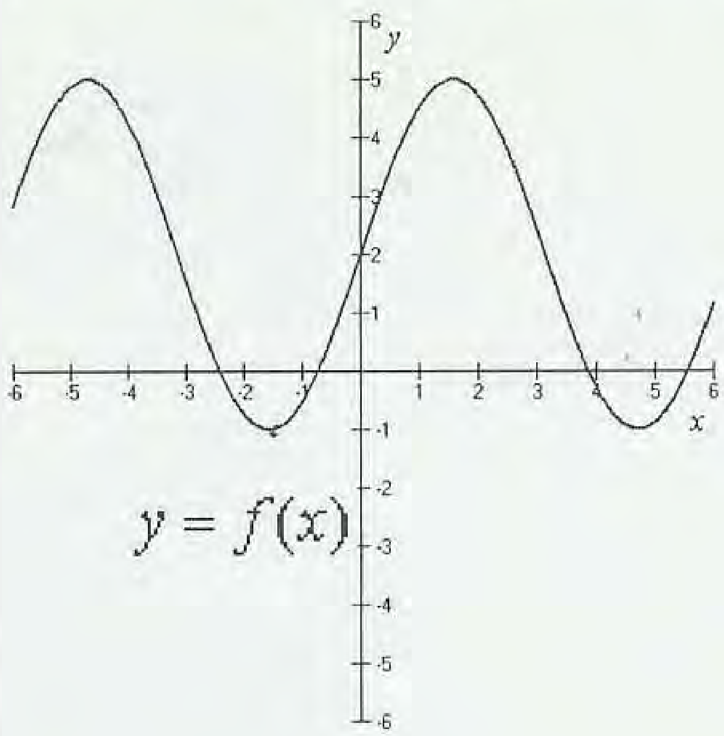
d) $\sin y=\cos\left(x^4\right)$, $\dfrac{dy}{dx}$

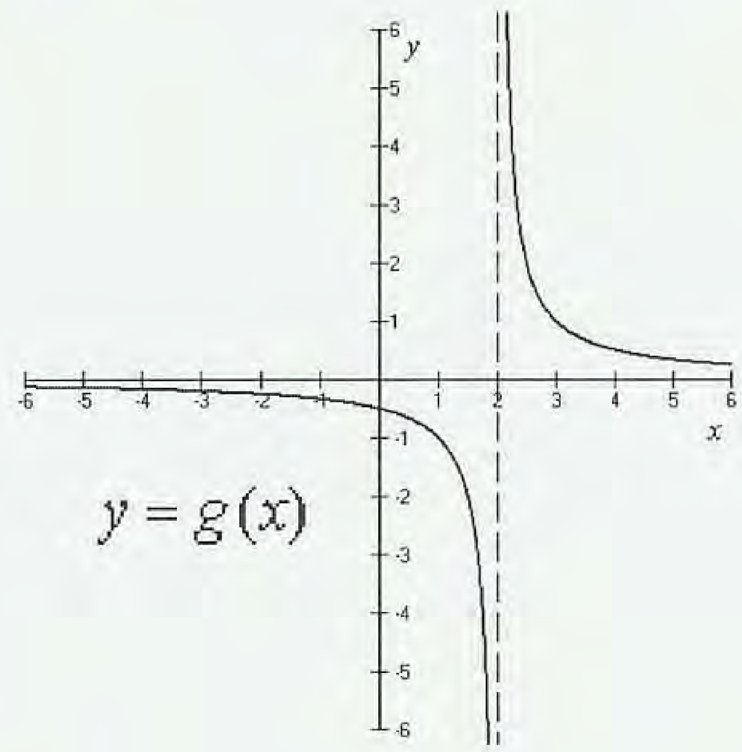
$\frac{d}{dx}(\sin y) = \frac{d}{dx}(\cos x^4)$

$\cos y \cdot \frac{dy}{dx} = -\sin x^4 \cdot 4x^3$

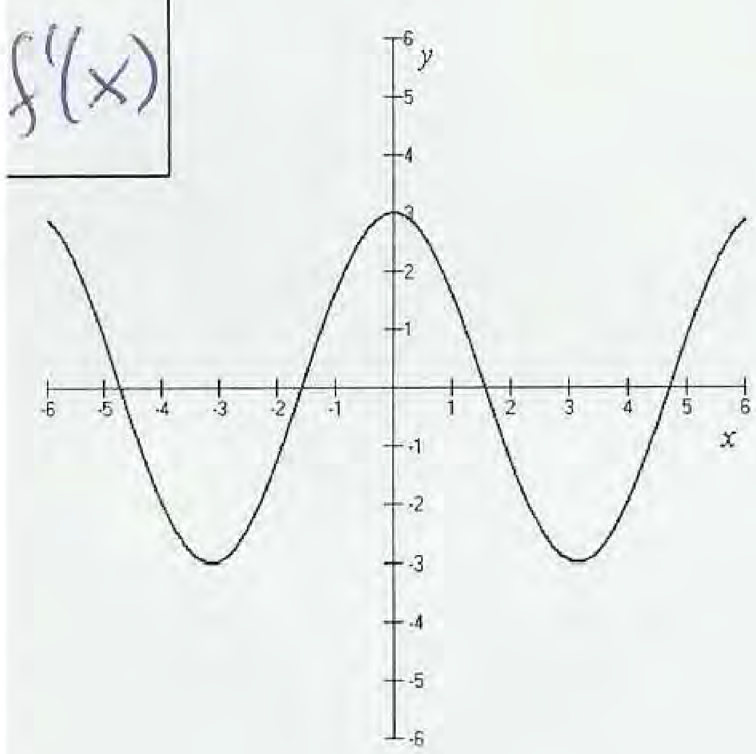
$\frac{dy}{dx} = \dfrac{-4x^3 \sin x^4}{\cos y}$

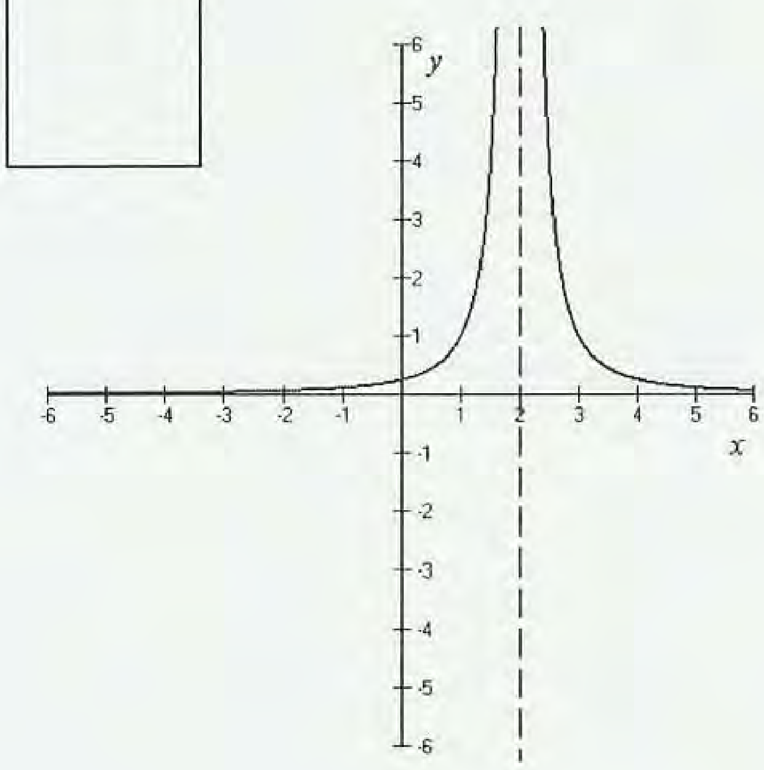
You are given the graphs of two functions $y = f(x)$ and $y = g(x)$ defined for all real numbers. Below are 4 further graphs. Identify those graphs that are the first derivative of *f* and *g* respectively and label the curves by placing $f'(x)$ a $g'(x)$ into the appropriate box. [**marks]**

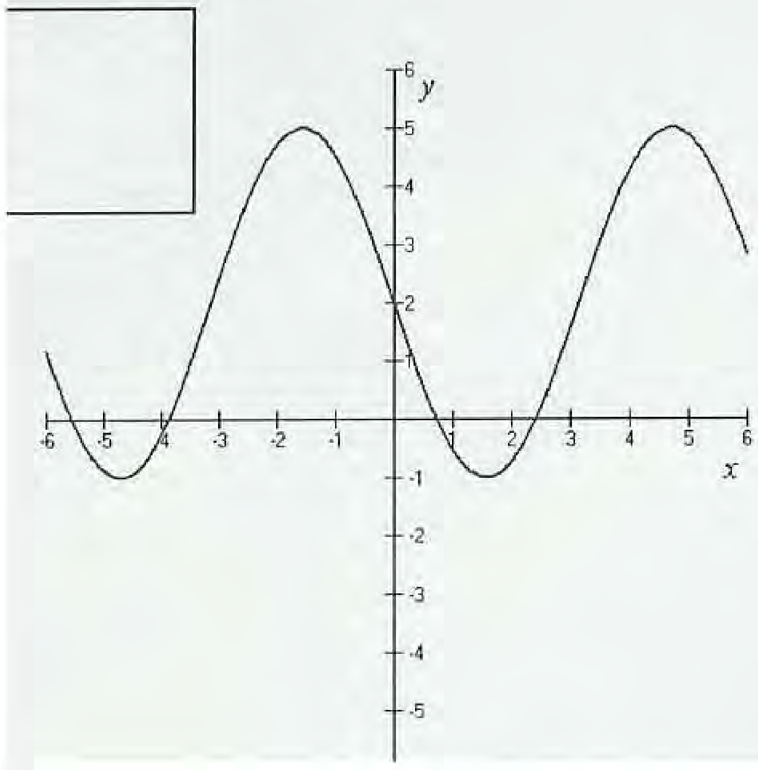




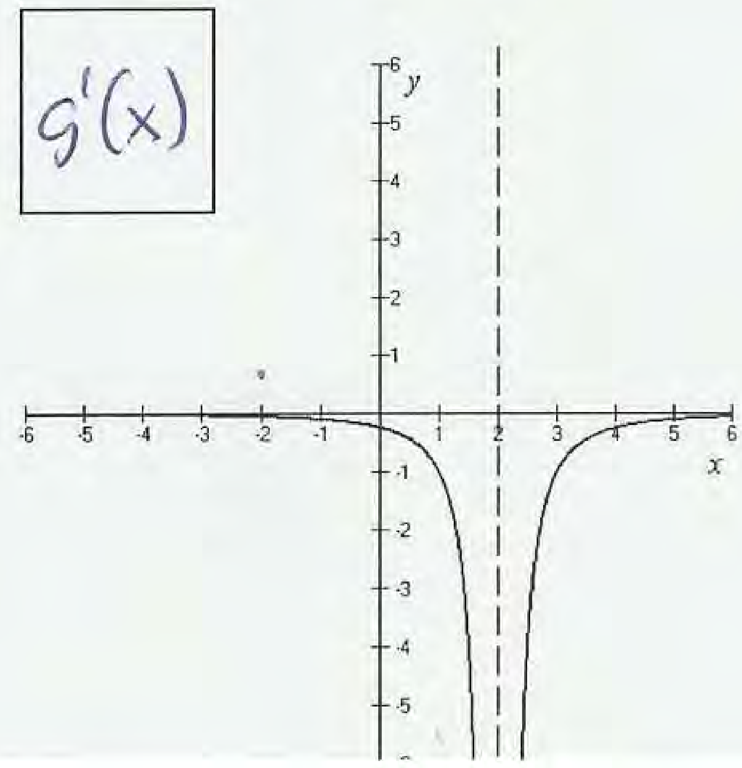
$f'(x)$







$g'(x)$



4. Approximate to two decimal places the value of $\sqrt{15.6}$ using linear approximation, differentials, or Newton-Raphson method. [**marks]**

linear approximation: $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

Let $y = f(x) = \sqrt{x}$ and $a = 16$. Then $f'(x) = \frac{1}{2\sqrt{x}}$,

$f(16) = \sqrt{16} = 4$, and $f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$.

$L(x) = 4 + \frac{1}{8}(x - 16)$.

$\sqrt{15.6} \approx L(15.6) = 4 + \frac{1}{8}(15.6 - 16) = 4 + \frac{1}{8}(-0.4) = 4 - \frac{1}{20} = 3\frac{19}{20}$

$\sqrt{15.6} \approx 3.95$

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differentials: Let $y = f(x) = \sqrt{x}$. Then $f'(x) = \frac{1}{2\sqrt{x}}$.

Take $x = 16$ changing to $x = 15.6$, so $\Delta x = 15.6 - 16 = -0.4$.

$\Delta y \approx dy = f'(x)\,\Delta x = f'(16)\cdot(-0.4) = \frac{1}{2\sqrt{16}}\cdot(-0.4) = -\frac{1}{20}$.

So, $\Delta y = \sqrt{15.6} - \sqrt{16} \approx -\frac{1}{20} \Rightarrow \sqrt{15.6} \approx \sqrt{16} - \frac{1}{20} = 4 - \frac{1}{20} = 3.95$

Newton-Raphson method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ given x_1.

Let $\sqrt{15.6} = x \Rightarrow 0 = x^2 - 15.6$.

Let $f(x) = x^2 - 15.6$, then $f'(x) = 2x$. Choose $x_1 = 4$.

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4 - \frac{4^2 - 15.6}{2\cdot 4} = 4 - \frac{0.4}{8} = 4 - \frac{1}{20} = 3.95$

So, $\sqrt{15.6} \approx 3.95$.

5. A certain property in Vancouver grows exponentially from being valued at \$250,000 in the year 2000 to \$400,000 in the year 2006. Let t measure time in years with $t = 0$ meaning year 2000. **[10 marks]**

a) Write an exponential function that models this.

Let $y(t)$ be the value of the property in dollars at time t

$y = y_0 e^{kt}$ with $y_0 = 250,000$ and $y(6) = 400,000$

Solve for k: $400,000 = 250,000\, e^{6k} \Rightarrow k = \frac{1}{6}\ln 1.6 \approx 0.$

$y = 250,000\, e^{\frac{t}{6}\ln 1.6} = 250,000\,(1.6)^{t/6}$

$(y \approx 250,000\, e^{0.0783t})$

b) What will be the approximate value of the property in 2010?

$2010 \Rightarrow t = 10$

$y(10) = 250,000\,(1.6)^{10/6} \approx \$547,192.30$

$(y(10) \approx \$547,006.63)$