Simon Fraser University
Department of Mathematics
Burnaby Campus

MATH 157-3, Summer 2008
Midterm 1
June 2nd, 2008, 11:30 – 12:20

Last Name (please print):	KEY (yellow)
First Name (please print):	
Student Number:	
Instructor:	P. Menz

Instructions:

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Fill in the above box.
3. This exam contains 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete.
4. SHOW ALL YOUR WORK!
5. If you run out of space in a problem, use the space on the back of the previous page and clearly indicate where the solution continues.
6. **Only** scientific, non-programmable calculators with no differentiation and integration capabilities are allowed.
7. No book, paper, or device, other than the usual writing instruments, this booklet and an acceptable calculator, shall be within reach of a student during the examination.
8. During the examination, speaking to, communicating with, or deliberately exposing written papers to the view of other examinees is forbidden.
9. Try your Best!

Do not write in this table!

Question	Marks
1	/4
2	/4
3	/6
4	/6
5	/6
6	/4
Total	**/30**

1. Short answer section: **[1 mark each = 4 marks]**

a) $|x| = \left(\sqrt{x}\right)^2$. If this is a true statement explain, or provide a counterexample.

False.

If $x < 0$, then $|x| > 0$ but $(\sqrt{x})^2$ does not exist.

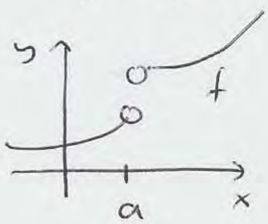
b) $\lim\limits_{x\to\infty} \dfrac{6x^m - 5x}{2x^n + 1} = \dfrac{6}{2} = 3$ for two positive integers m and n with $m > n$. If this is a true statement explain, or provide a counterexample.

Choose $m = 2$, $n = 1$.

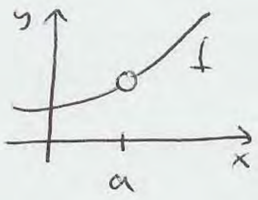
$\lim\limits_{x\to\infty} \dfrac{6x^2 - 5x}{2x+1} = \infty$ since deg. (num) > deg. (denom)

c) Name two types of discontinuities for a function f at $x = a$ and provide a sketch for each.

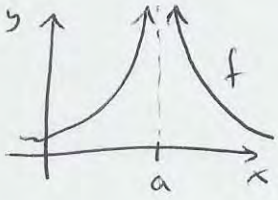
jump



removable

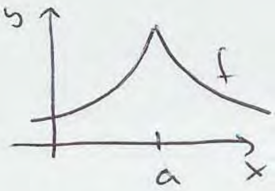


infinite

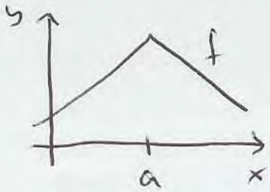


d) Name one condition under which a function f is not differentiable at $x = a$ and provide a sketch.

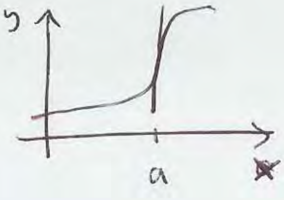
cusp



corner



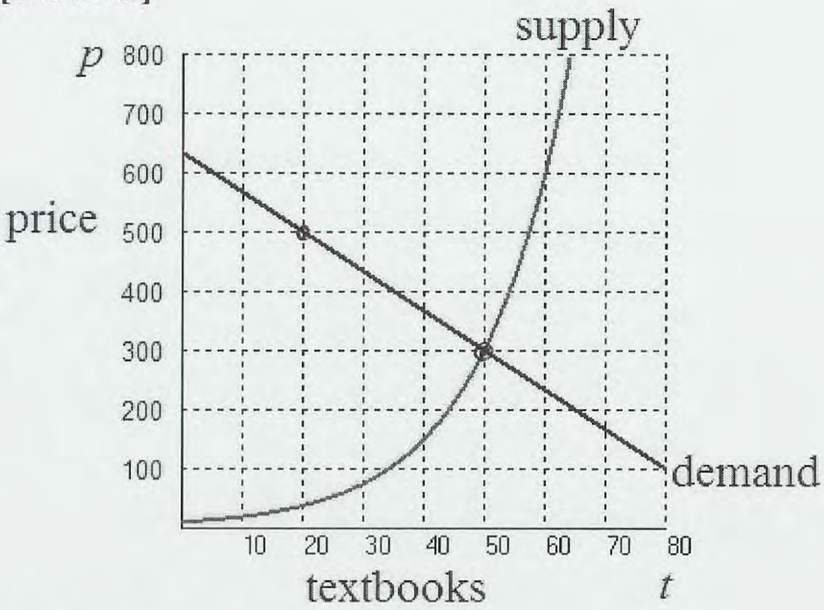
vertical tangent



discontinuity

see c)

2. The graphs of the supply and demand equations for calculus textbooks is given below, where the price p is in dollars and t is the number of textbooks.
[4 marks]



a) What is the demand equation?

points: $(20, 500)$, $(50, 300)$

$$m = \frac{500-300}{20-50} = -\frac{200}{30} = -\frac{20}{3}$$

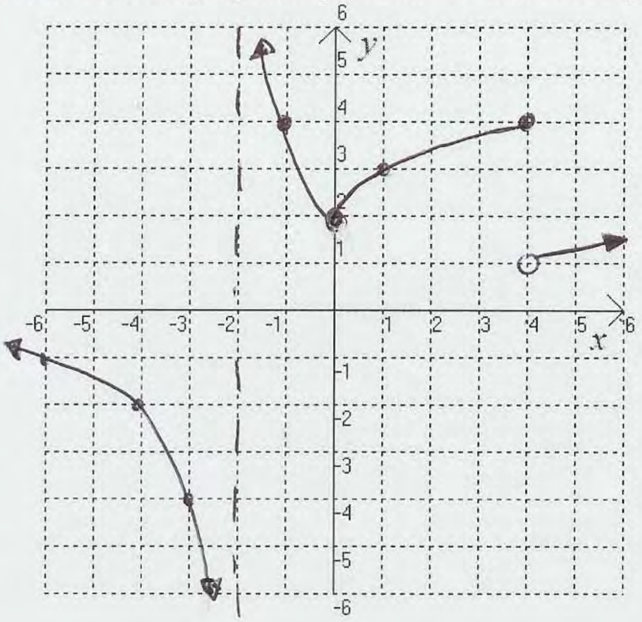
$$p - 500 = -\frac{20}{3}(t-20) \quad \text{OR} \quad p = -\frac{20}{3}t + \frac{1900}{3}$$

b) What are the equilibrium price and quantity?

$300   50

3. Let $f(x)=\begin{cases} \dfrac{4}{x+2}, & x<0 \\ \sqrt{x}+2, & 0\le x\le 4 \\ \dfrac{1}{4}x, & x>4 \end{cases}$. **[6 marks]**

a) Sketch the graph of f in the coordinate system below.



b) What is the range of f?

$(-\infty, 0) \cup (1, \infty)$

c) Is f continuous at $x = 4$? Why or why not?

jump discontinuity at $x = 4$ from graph

OR

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (\sqrt{x}+2) = 4$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \frac{1}{4}x = 1$

$\neq$ Therefore discontinuous at $x = 4$.

4. Find the indicated derivative provided it exists. Do not simplify. **[6 marks]**

a) $y = (x^4 + x^3 + x^2 + x + 1)(1 - x)$, y'

$$y' = (4x^3 + 3x^2 + 2x + 1)(1-x) - (x^4 + x^3 + x^2 + x + 1)$$

b) $y = \dfrac{x}{x-7}$, $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1(x-7) - x(1)}{(x-7)^2} = \frac{-7}{(x-7)^2}$$

c) $h(x) = \sqrt[3]{x^4 - 3x}$, $h'(x)$

$$h'(x) = \frac{1}{3}(x^4 - 3x)^{-\frac{2}{3}}(4x^3 - 3)$$

5. Describe the following limits in terms of a real number, $\pm\infty$, or **does not exist**.
[6 marks]

a) $\lim_{x\to 2} \frac{3}{x^2-4}$

$$\left.\begin{array}{l} \lim_{x\to 2^+} \frac{3}{x^2-4} \overset{\frac{+\#}{+0}}{=} \infty \\ \lim_{x\to 2^-} \frac{3}{x^2-4} \overset{\frac{+\#}{-0}}{=} -\infty \end{array}\right\} \Rightarrow \lim_{x\to 2} \frac{3}{x^2-4} \text{ DNE}$$

b) $\lim_{x\to -5^-} \frac{|x+5|}{x+5} = \lim_{x\to -5^-} \frac{-(x+5)}{x+5} = \lim_{x\to -5^-} (-1) = -1$

c) $\lim_{x\to 3} \frac{9-x^2}{x-3} = \lim_{x\to 3} \frac{(3-x)(3+x)}{x-3} = \lim_{x\to 3} -(3+x) = -6$

6. The profit in dollars from selling x pounds of cocoa beans is given by $P(x) = 30\sqrt{x} + 1$. **[4 marks]**

   a) Find the average rate of change in profit from selling 4 lb to 9 lb of cocoa beans.

$$P_{avg} = \frac{P(9) - P(4)}{9-4} = \frac{(30\sqrt{9}+1)-(30\sqrt{4}+1)}{5}$$

$$= \frac{90+1-60-1}{5} = \frac{30}{5} = 6 \ \$/lb$$

   b) Find the marginal profit ~~from selling 9 lb of cocoa beans~~. You must differentiate by using the definition of the derivative.

$$P'(x) = \lim_{h \to 0} \frac{P(x+h) - P(x)}{h}$$

$$= \lim_{h \to 0} \frac{(30\sqrt{x+h}+1) - (30\sqrt{x}+1)}{h}$$

$$= \lim_{h \to 0} \frac{30(\sqrt{x+h} - \sqrt{x})}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h \to 0} \frac{30(x+h-x)}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h \to 0} \frac{30h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h \to 0} \frac{30}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{30}{2\sqrt{x}}$$