MATH 157 **Instructor: R. Pyke**

Final Exam 2008

Last Name:	
First Name:	
SFU Student email :	`@sfu.ca`

1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.
5. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

Question	Score	Max		Question	Score	Max
1		7		9		6
2		5		10		5
3		4		11		7
4		5		12		4
5		13		13		4
6		5		14		3
7		6		15		6
8		7		16		17
				Total		104

(1) [Marks: 7] Determine the following limits as either a number, $\pm\infty$, or DNE (does not exist). If DNE explain why it doesn't exist.

(a) $\displaystyle\lim_{x\to-\infty} \frac{4x^3 - 5x - 8}{\sqrt{2x^6 + 5}}$

(b) $\displaystyle\lim_{x\to 0} \frac{\sqrt{2x+1} \ - \ 1}{3x}$

(c) $\displaystyle\lim_{x\to 2/3} \frac{|2 - 3x|}{2x - \frac{4}{3}}$

(2) [Marks: 5] Consider the function

$$f(x) \;=\; \begin{cases} k\sqrt{x+7} & x < 2 \\ c & x = 2 \\ 2x - k & x > 2 \end{cases}$$

Find a value of k and c which makes the function f continuous on $(-\infty, \infty)$. Explain your reasoning.

(3) [Marks: 4] On average, a factory needs 100 kilowatts (kW) of electricity but the actual amount of electricity needed varies according to the demand of production at that moment. The amount of electricity, in kW, needed in addition to this 100 kW is given by

$$q(t) = \frac{1}{2}t^3 + \frac{1}{10}t^2 - 2t + 1$$

where t is days and $t = 0$ corresponds to June 1 of this year (so, if $q(t) > 0$ the factory is using more than 100 kW of electricity at this time, and if $q(t) < 0$ it is using less than 100 kW).

Use the Intermediate Value Theorem to find a day (that is, an interval in t of length less than or equal to 1) during the year when the factory was using 105 kW of electricity.

(4) [Marks: 5] The fee charged for parking a car at a downtown lot is as follows. \$1 as soon as you arrive, then increasing linearly from \$1 to \$2 during the next hour. At the beginning of the second hour another dollar is added to the price at that point (so, it is now \$3), then, again, the price increases linearly by a dollar over the next hour. At the beginning of the third hour a dollar is added to the price and then the price increases linearly by a dollar over the next hour. This same pattern is followed for all subsequent hours; the price increases linearly by a dollar for the hour, and at the start of each hour a dollar is added to the price at that point.

(a) What price will you pay if you parked for 4 hours and 30 minutes?

(a) Make a sketch of the graph of the price $P(t)$ of parking as a function of time over the first 6 hour period.

(b) Identify any points of discontinuity of $P(t)$ and state what type of discontinuity it is and why it is that type.

(5) [Marks: 13] Find the indicated derivatives. You do not need to simplify your answer.

(a) $f(x) = \dfrac{x^2(x^2+1)}{x^2-1}$, $f\,'(x)$

(b) $g(t) = \sqrt[3]{7-3t^3} \; + \; t^{5/7}$, $g\,'(t)$

(c) $h(x) = 5^x \; - \; \log_3(2x)$, $h\,'(x)$

(d) $z = \tan^{-1}(e^t)$, z'

(e) $P(q) = 2\sin(\cos 3q)$, $P'(q)$

(f) $3y^3 - 2x^2 + xy = 3$, y''

(6) [Marks: 5] Use the definition of the derivative (**not** the rules of differentiation) to find the slope of the tangent line to the graph of $f(x)$ at the point $(3,\ f(3))$, where

$$f(x) = \frac{1}{\sqrt{x} - 1}$$

(7) [Marks: 6] The occupancy of a hotel (that is, the number of guests in the hotel) is given by

$$R(t) = 60 + 32\sin^2\left(\frac{\pi t}{12}\right), \qquad 0 \le t \le 12$$

where t is measured in months and $t = 0$ corresponds to the beginning of January.

(a) What is the occupancy at the beginning of May?

(b) What is the marginal occupancy at the beginning of May?

(c) Find a date t <u>during the months of May - Aug</u> when the marginal occupancy is not changing. What is the significance of this date for the occupancy?

(8) [Marks: 7] The demand equation for a certain commodity is

$$p = \sqrt[3]{1000 - x}$$

where p is the price in dollars and x is the number demanded.

(a) Determine the range of *prices* corresponding to inelastic, unitary, and elastic demand.

(b) For what value of the *demand* is the demand unitary?

(c) If a price of \$6 is increased by $\frac{1}{2}\%$, what is the approximate percentage change in demand? Will revenue increase or decrease?

(9) [Marks: 6] Recall that market equilibrium prevails when the quantity produced is equal to the quantity demanded. If the supply equation is $p = 2x^2 - 1$ and the demand equation is $p = 8 + \dfrac{3}{x^2+1}$, use the Newton-Raphson method to estimate the market equilibrium. Begin at 2 and perform enough iterations until you know that your estimate is accurate to two decimal places.

(10) [Marks: 5] Sales per day in Canada of a particular drug can be modelled by

$$Q(t) \;=\; \frac{3000}{1 + 420e^{-kt}}$$

measured in millions of dollars, where t is the number of days after the drug is first introduced to the market.

(a) If sales were 124 million dollars on the fifth day after the drug first appears on the market, what will the sales be on the tenth day?

(b) Approximately, what will be the sales per day *many days* after the drug is introduced? Is this an overestimate or an underestimate? (that is, is your estimate greater than or less than the actual number of sales after many days?)

(11) [Marks: 7] The demand equation for a certain kind of battery is $11x^2 + 2p^2 = 4400$ where x represents the number (in thousands) of batteries demanded each week when the unit price is p dollars.

Suppose the demand, in thousands, changes in time according to the equation

$$x(t) = 2.2 + 0.4\sqrt{2t}$$

where t is days from January 1.

(a) What is the quantity demanded 100 days after January 1?

(b) What is the price of the battery 100 days after January 1?

(c) How quickly is the quantity demanded changing 100 days after January 1?

continued ⟶

(d) How quickly is the price changing 100 days after January 1?

(e) How quickly is the price changing when the quantity demanded is 8,000?

(12) [Marks: 4] A manufacturer of belts finds that the total cost $C(x)$ (in dollars) of manufacturing x belts per day is given by $C(x) = 400 + 4x + 0.0001x^2$. Each belt can be sold at a price of p dollars, where p is related to x by the demand equation $p = 10 - 0.0004x$.

If all belts that are manufactured can be sold, what price should the belts be sold at so that the largest profit is attained?

(13) [Marks: 4] **(a)** A bank account pays interest at the rate 4.2% per annum compounded semiannually. $2,000 is deposited today. When will the account reach $10,000?

(b) Bank A gives 6% interest compounded semiannually. Bank B provides the same return per year on your money but advertises an interest rate that is compounded monthly. What interest rate (stated nominally) does Bank B provide?

(14) [Marks: 3] You plan to buy a car in 5 years (from the beginning of next month) that will cost $\$25,000$ at that time. How much should you deposit into an account at the end of each month (beginning at the end of next month) that pays 7.25% interest compounded monthly so that you can purchase the car at that time?

(15) [Marks: 6] You are renovating your house and require a new roof which costs $12,500. You make a down payment of $3,300, the remaining amount is amortized with 48 equal monthly payments at 5.15% per annum compounded monthly.

(a) What is the amount of each payment?

(b) What is the total amount paid for the roof?

(c) What is the total amount of interest paid?

(d) What amount is still owed after the 30th payment?

(16) [Marks: 17] Sketch the graph of the function $h(x) = \dfrac{x+1}{x^2-2x-1}$ by completing the following steps;

(a) What is the domain of $h(x)$?

(b) What are the x and y intercepts?

continued $\longrightarrow$

(c) Are there any horizontal or vertical asymptotes? If so, determine here how the graph approaches the asymptote (if there's a horizontal asymptote does the graph approach from the top or bottom, and if there's a vertical asymptote does the graph approach $+\infty$ or $-\infty$ and from which side of the asymptote?).

continued $\longrightarrow$

(d) Given that $h'(x) = -\dfrac{x^2+2x-1}{(x^2-2x-1)^2}$, find critical points and regions where $h(x)$ is increasing or decreasing (make a sign diagram).

(e) Classify each of the critical points as either local maxima, local minima, or neither by using the first derivative.

continued ⟶

(f) Given that $h''(x) = \dfrac{4(x^2+1)}{(x^2-2x-1)^3}$, find any points of inflection and the regions of concavity.

(g) Apply the *second derivative* test to the critical points and state the result as either local maximum, minimum, or undecidable.

(h) Using all this information make a sketch of the graph of $h(x)$ on the next page

END