

## MATH 155 Midterm Exam 2

Last Name:\_\_\_\_\_

Given Name(s):\_\_\_\_\_

Student ID #:\_\_\_\_\_

Signature:\_\_\_\_\_

### INSTRUCTIONS:

1. Print your name and ID # in the spaces given.
2. Sign your name at the indicated place.
3. This exam has 5 questions on Page 2 to Page 6.
3. The allocated time for this exam is 50 minutes.
4. Calculators are not allowed.
5. A formula sheet is attached.
6. Answers to all questions have to be properly justified.

Questions	Q1	Q2	Q3	Q4	Q5	Total
Maximum	20	20	20	20	20	100
Your Score						

**Q1.** Identify each integral as improper or proper; Then in the case of improper integral, determine whether it is convergent or divergent.

(a)  $\int_0^4 \frac{1}{\sqrt{x}} dx$

(b)  $\int_0^1 \sqrt{1-x} dx$

(c)  $\int_{-\infty}^0 xe^x dx$

**Q2.** Suppose the Midpoint Rule, the Trapezoidal Rule and Simpson's Rule with  $n = 10$  are to be used to approximate the integral  $\int_0^1 e^{x^2} dx$ .

(a) Find the approximate expression in each case.

(b) In terms of accuracy, which rule do you expect to perform best? Which is worst? Given the facts that  $\max_{0 \leq x \leq 1} |f^{(2)}(x)| \leq 6e$  and  $\max_{0 \leq x \leq 1} |f^{(4)}(x)| \leq 76e$  where  $f(x) = e^{x^2}$ , justify your answer.

**Q3.** Solve the following initial value problem  $\begin{cases} \frac{dy}{dt} = y - y^2 \\ y(0) = 10 \end{cases}$

**Q4.** A system is given by  $\frac{dy}{dt} = y(2y - 1)(y + 2)$ .

(a) Find all the equilibria of this system.

(b) Use the eigenvalue approach to determine their stability.

(c) Use the graphical approach to explain your results in (b).

**Q5.** Solve the following linear systems:

$$(a) \quad \begin{cases} 2x + y - z = 1 \\ 4x - 2y + 2z = 6 \end{cases}$$

$$(b) \quad \begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ 3x + y - 2z = -1 \end{cases}$$