

SIMON FRASER UNIVERSITY  
DEPARTMENT OF MATHEMATICS  
Instructor: A. Wise

Midterm 2

MATH 155

July 14th, 2003, 8:30-9:20am

Name: \_\_\_\_\_ (please print)  
*family name* *given name*

Signature: \_\_\_\_\_

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Write your name above in block letters and sign below your name.  
Write your student number and your family name in the boxes on the back page.
3. For each question write your final answer in the box when one is provided. You must show all your work when space is provided. If this space is insufficient you may use the back of the previous page.
4. This exam contains this cover page, the normal distribution table, two formula sheets and 7 pages with a total of 6 questions. Once the exam begins please check to make sure your exam is complete with a total of 11 pages.
5. No book, paper, or device, other than the usual writing instruments and this booklet, shall be within reach of a student during the examination. In particular, **no calculators are allowed.**
6. **During the examination, speaking to, communicating with, or exposing written papers to the view of, other examinees is forbidden.**
7. **Students observed writing anything after the call to stop writing will be subject to summary penalties.**

- [5] 1. Solve the initial value problem

$$\frac{dy}{dx} = (y+1)^3, \text{ with } y(0) = 1.$$

ANSWER

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SHOW YOUR WORK

2. Suppose we are studying a given species in a metapopulation using Levin's model. That is, the fraction of patches occupied by the species at time  $t$ ,  $p(t)$ , satisfies

$$\frac{dp}{dt} = 2p(1-p) - p \text{ for } t \geq 0.$$

- [4] (a) Find all equilibria of this model and discuss their stability **graphically** or using the **eigenvalue method**.

ANSWER

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SHOW YOUR WORK

- [3] (b) Find  $\lim_{t \rightarrow \infty} p(t)$  if  $p(0) = p_0 > 0$  using your result in part (a) **without actually solving the differential equation**. Will the species survive or be extinct in the long run? Provide a clear answer and explanation below.

ANSWER

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ROUGH WORK for finding  $\lim_{t \rightarrow \infty} p(t)$ :

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Will the species survive or be extinct in the long run? Explain.

- [4] (c) What is the condition on  $c$  and  $m$  which makes the differential equation  $\frac{dp}{dt} = cp(1-p) - mp$  have only one equilibrium? What is the equilibrium in this case and is it stable? Explain.

ANSWER

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Condition(s) on the constants  $c$  and  $m$ :

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Equilibrium when above condition(s) hold:

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Is this equilibrium stable or unstable? Explain.

- [3] 3. (a) Find the Taylor Polynomial of degree 3 about  $x = 0$  for  $f(x) = \sin x$ .

ANSWER

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SHOW YOUR WORK

- [2] (b) Use your result in part (a) to approximate the value of  $\frac{1}{\sqrt{2}}$ . Leave the answer in **unsimplified numerical form**.

ANSWER

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SHOW YOUR WORK

- [3] (c) Give an upper bound for the error of your approximation using the formula

$$|R_n(x)| = \frac{|f^{(n+1)}(z)|}{(n+1)!} |x - a|^{n+1}.$$

Leave the answer in **unsimplified numerical form**

ANSWER

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SHOW YOUR WORK

- [5] 4. Suppose a variable  $X$  is normally distributed with mean 4 and standard deviation 2. Find the probability that the variable  $X$  assumes values greater than 2.

ANSWER
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SHOW YOUR WORK:

5. Suppose that a quantity  $X$  is distributed according to the density function

$$f(x) = \begin{cases} (a-1)x^{-a} & \text{if } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

where  $a$  is a constant greater than 2.

- [4] (a) Find the mean of  $X$ . Your answer should be in terms of the constant  $a$ .

ANSWER

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SHOW YOUR WORK

- [2] (b) Set up the integral equation which will find the value of  $x$  such that  $P(X < x) = 0.5$ . Do not compute this integral or solve this equation.

ANSWER

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SHOW YOUR WORK

- [5] 6. State the reason(s) why the integral  $\int_{-\pi/2}^{\pi/2} \tan x \, dx$  is considered **improper**. Does this integral **converge or diverge**? If it diverges explain why. If the integral converges find the value that it converges to. (You may refer to the formula sheet to obtain  $\int \tan x \, dx$ .)

ANSWER
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SHOW YOUR WORK:

Student number

Family name

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DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	5	
2	11	
3	8	
4	5	
5	6	
6	5	
Total	40	