ORANGE AND GREEN PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 2

7 March 2007, 08:30–09:20

Last Name _____

Given Name(s) _____

Student # _____

Signature _____

INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Write your last name, given name(s), and student number in the box above. Sign on the last line of the box.
3. This exam has 6 questions on 6 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.**
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

| Question | Maximum | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 6 | |
| 3 | 10 | |
| 4 | 6 | |
| 5 | 8 | |
| 6 | 9 | |
| Total | 45 | |

MATH 155

[6] 1. Evaluate $\int \frac{3x-1}{x^2-2x-3}\,dx$.

$x^2-2x-3=(x-3)(x+1)$ by solving $x^2-2x-3=0$

$x_{1,2}=\frac{2\pm\sqrt{4+12}}{2}$

$$\frac{3x-1}{x^2-2x-3}=\frac{A}{x-3}+\frac{B}{x+1}=\frac{A(x+1)+B(x-3)}{(x-3)(x+1)}=$$

$$=\frac{(A+B)x+(A-3B)}{(x-3)(x+1)}$$

$$\left.\begin{aligned}A+B&=3\\A-3B&=-1\end{aligned}\right\}\Rightarrow B=3-A$$

$A-3(3-A)=-1 \Rightarrow 4A=8$

$\boxed{\begin{aligned}A&=2\\B&=1\end{aligned}}$

$$\int\frac{3x-1}{x^2-2x-3}\,dx=\int\left[\frac{2}{x-3}+\frac{1}{x+1}\right]dx=$$

$$=2\ln|x-3|+\ln|x+1|+C$$

[6] 2. Evaluate $\int \frac{x^2+6x+5}{x^2+4}\,dx.$

$$\int \frac{x^2+6x+5}{x^2+4}\,dx = \int \left[1 + \frac{6x+1}{x^2+4}\right] dx =$$

$$= x + \int \frac{1}{4} \cdot \frac{6x+1}{\frac{x^2}{4}+1}\,dx = \quad \boxed{\begin{array}{l} u = \frac{x}{2} \\ dx = 2\,du \end{array}}$$

$$= x + \frac{1}{4}\int \frac{12u+1}{u^2+1}\,2\,du =$$

$$= x + 3\int \frac{2u}{u^2+1}\,du + \frac{1}{2}\int \frac{1}{u^2+1}\,du =$$

$$= x + 3\ln|u^2+1| + \frac{1}{2}\tan^{-1} u + C =$$

$$= x + 3\ln\left|\left(\frac{x}{2}\right)^2+1\right| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

Another form of the correct answer is

$$x + 3\ln|x^2+4| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C.$$

[5] **3.** (a) Use integration by parts to find $\int \ln x \, dx$. (Hint: Multiply by 1.)

$u = \ln x \quad v' = 1 \quad v = x$

$$\int \ln x \, dx = x \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - x + C$$

[5] (b) Explain why the integral

$$\int_0^1 \ln x \, dx$$

is improper. Evaluate this integral. (Use the result of part (a).)

The integral is improper because $\lim_{x \to 0^+} \ln x = -\infty$.

$$\int_0^1 \ln x \, dx = \lim_{c \to 0^+} \int_c^1 \ln x \, dx \overset{(a)}{=}$$

$$= \lim_{c \to 0^+} (1 \cdot \ln 1 - 1 - c \ln c + c) =$$

L'Hospital's Rule

$$= -1 - \lim_{c \to 0^+} c \ln c = -1 - \lim_{c \to 0^+} \frac{\ln c}{\frac{1}{c}} =$$

$$= -1 - \lim_{c \to 0^+} \frac{\frac{1}{c}}{-\frac{1}{c^2}} = -1 - \lim_{c \to 0^+} (-c) = -1$$

[6] 4. Use the trapezoidal rule with $n = 4$ intervals to find an approximation for $\int_0^1 \sqrt{x}\,dx$. Compare the approximation with the exact value of the integral.

$$\Delta x = \frac{1-0}{4} = 0.25$$

$$x_0 = 0,\quad x_1 = 0.25,\quad x_2 = 0.5,\quad x_3 = 0.75,\quad x_4 = 1$$

$$\int_0^1 \sqrt{x}\,dx \approx \frac{1-0}{4}\cdot\left(\frac{\sqrt{0}}{2} + \sqrt{0.25} + \sqrt{0.5} + \sqrt{0.75} + \frac{\sqrt{1}}{2}\right)$$

$$\approx 0.25\cdot(0 + 0.5 + 0.7071 + 0.8661 + 0.5)$$

$$\approx 0.6433$$

The exact value is

$$\int_0^1 \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3} \approx 0.66667.$$

[6] **5.** (a) Compute the Taylor polynomial of degree 5 about $x = 0$ for the function $f(x) = \sin(x)$.

$f(x) = \sin x \qquad f(0) = 0$

$f'(x) = \cos x \qquad f'(0) = 1$

$f''(x) = -\sin x \qquad f''(0) = 0$

$f'''(x) = -\cos x \qquad f'''(0) = -1$

$f^{(4)}(x) = \sin x \qquad f^{(4)}(0) = 0$

$f^{(5)}(x) = \cos x \qquad f^{(5)}(0) = 1$

$$P_5(x) = x + \frac{-1}{3!}x^3 + \frac{1}{5!}x^5 =$$

$$= x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

[2] (b) Use the polynomial found in part (a) to approximate the value $\sin(1.3)$. Compare this approximation with the exact value of $\sin(1.3)$.

$$P_5(1.3) = 1.3 - \frac{(1.3)^3}{6} + \frac{(1.3)^5}{120} \approx 0.9648$$

$$\sin(1.3) \approx 0.9636$$