

MATH 155 Midterm Exam 1

Last Name:_____

Given Name(s):_____

Student ID #:_____

Signature:_____

INSTRUCTIONS:

1. Print your name and ID # in the spaces given.
2. Sign your name at the indicated place.
3. This exam has 5 questions on Page 2 to Page 5.
3. The allocated time for this exam is 50 minutes.
4. Calculators are not allowed.
5. A formula sheet is attached.
6. Answers to all questions, except Q1, have to be properly justified.

Questions	Q1	Q2	Q3	Q4	Q5	Total
Maximum	20	40	10	15	15	100
Your Score						

Q1. Express the following quantities in terms of definite integrals. Do not evaluate the integrals. (4 marks for each)

(a) The area A enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$.

(b) The displacement d of a particle over time interval $[1, 4]$. Assume it is moving along a straight line with velocity $v(t) = -t^2 + 2t$

(c) The population $p(t)$ of a certain species with growth rate $r(t) = e^{-5t}$ and initial population $p(0) = 10000$

(d) The volume of a solid obtained by rotating around x-axis the area enclosed by $y = 4 - x^2$ and $y = 0$.

(e) The average value of the function $f(x) = \sqrt{1 + x^4}$ on $[0, 2]$

Q2. Evaluate the following integrals. (8 marks for each)

(a) $\int_0^1 \sqrt{x}(1-x) dx$

(b) $\int \frac{x}{\sqrt{1+x^2}} dx$

(c) $\int_{-1}^2 e^{-|s|} ds$

(d) $\int x^{2004} \ln x dx$

(e) $\int \cos \sqrt{t} dt$

Q3. (10 marks) Give the partial fraction decomposition of

$$f(x) = \frac{x+2}{x^3(x+1)^2(x^2-x+1)^2}$$

Do not determine the coefficients.

Q4. (a) (6 marks) Show that $\int_1^2 e^{x^2} dx \leq \int_1^2 e^{x^3} dx$

(b) (9 marks) Show that $12 \leq \int_0^4 \sqrt{9+x^2} dx \leq 20$

Q5. The limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n e^{\frac{3i}{n}} \cdot \frac{1}{n}$ is the definite integral of a certain function over $[0, 1]$. That is

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n e^{\frac{3i}{n}} \cdot \frac{1}{n} = \int_0^1 f(x) dx$$

for some $f(x)$, where equal subintervals are used and the right endpoint of each interval chosen.

(a) (6 marks) Identify the function $f(x)$.

(b) (9 marks) Based on information from (a), evaluate the given limit.