BLUE AND ORANGE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 1

7 February 2007, 08:30–09:20

Last Name ____________________

Given Name(s) ____________________

Student # ____________________

Signature ____________________

INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Write your last name, given name(s), and student number in the box above. Sign on the last line of the box.
3. This exam has 6 questions on 5 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing instruments, this examination booklet, and a scientific calculator are allowed. **Calculators with graphing and/or symbolic computation capabilities are not allowed.**
5. **During the examination, speaking to, communicating with, or exposing written papers to the view of other examinees is forbidden.**
6. You may use the **reverse side of the previous page** for rough work or if you run out of space.
7. **You may lose marks if your explanations are incomplete or poorly presented.**
8. **Stop writing when you are instructed to do so. Failure to follow instructions may result in penalties.**

Question	Maximum	Score
1	7	
2	8	
3	5	
4	5	
5	6	
6	8	
Total	39	

1. Clearly indicate if the following statements are true (T) or false (F).

Assume that all functions are continuous in the intervals of integration.

A statement containing general constants and/or functions (f, g, a, b, c, n) is true if and only if it holds for *all admissible choices* of these constants and/or functions.

[1] (a) T $\sum_{k=1}^{n}(k+1) = \frac{n(n+3)}{2}$ $= \frac{n(n+1)}{2} + n$

[1] (b) F $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right) \cdot \left(\int_a^b g(x)\,dx\right)$ $f(x) = 1$

[1] (c) F If $\int_a^b f(x)\,dx \geq 0$, then $f(x) \geq 0$ for all x in $[a, b]$.

[1] (d) F If $f(x) \geq c$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq c$.

[1] (e) T $\int_0^1 x \sin x\,dx \leq \frac{1}{2}$ $\sin x \leqq 1 \Rightarrow \int_0^1 x \sin x\,dx \leqq \int_0^1 x\,dx = \frac{1}{2}$

[1] (f) T $\int_a^b [f(x) - g(x)]\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$

[1] (g) T $\int_a^b f(x)\,dx = \left(\int_a^c f(x)\,dx\right) - \left(\int_b^c f(x)\,dx\right)$

Explanations <u>not</u> required.

[5] **2.** (a) Let $L(x)$ denote the length of a certain organism at age x ($x \geq 0$). At the moment of birth (age $x = 0$) the length of the organism was $L(0) = 5$. By studying literature you learned that $L(x)$ is governed by

$$\frac{dL}{dx} = e^{-x/10}.$$

Find $L(x)$ for each $x \geq 0$.

$$L(x) = -10e^{-\frac{x}{10}} + C$$

$$L(0) = 5 \qquad -10e^0 + C = 5$$

$$-10 + C = 5$$

$$C = 15$$

$$\boxed{L(x) = -10e^{-\frac{x}{10}} + 15}$$

[3] (b) There is a certain length L_∞ which the organism will approach at maturity (at very large age). Determine L_∞.

$$L_\infty = \lim_{x \to \infty} \left(-10e^{-\frac{x}{10}} + 15\right) =$$

$$= \left(-10 \lim_{x \to \infty} e^{-\frac{x}{10}}\right) + 15 = -10 \cdot 0 + 15$$

$$\boxed{L_\infty = 15}$$

[5] **3.** Determine the average value of $f(x) = \dfrac{3}{\sqrt{1-x^2}}$ in the interval $[0, 1/2]$.

The average value is

$$f_{avg} = \frac{1}{\frac{1}{2} - 0} \cdot \int_0^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx$$

$$= 2 \cdot 3 \cdot \left[\sin^{-1} x\right]_0^{1/2} = 6 \cdot \left(\frac{\pi}{6} - 0\right) = \pi$$

$$\boxed{f_{avg} = \pi}$$

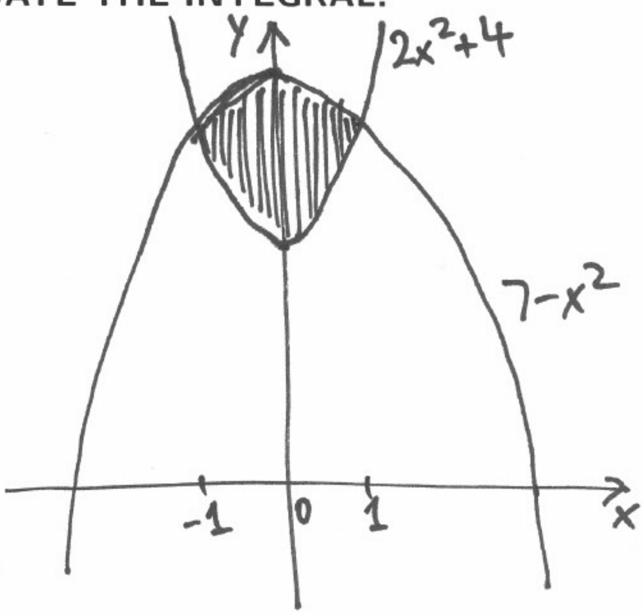
[5] **4.** Express the area between the curves $y = 2x^2 + 4$ and $y = 7 - x^2$ as a definite integral. **DO NOT EVALUATE THE INTEGRAL.**

$2x^2 + 4 = 7 - x^2$

$3x^2 = 3$

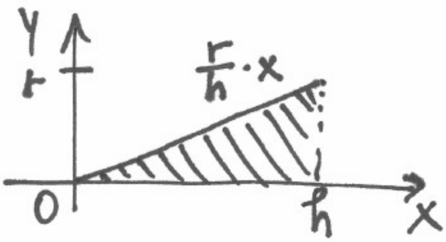
$x^2 = 1$

$x = \pm 1$



$$\text{Area} = \int_{-1}^{1} \left[(7 - x^2) - (2x^2 + 4)\right] dx$$

[3] **5.** (a) Describe the right-circular cone with base radius r and height h as a solid of revolution.



The solid is obtained by rotating the triangle drawn in the picture about the x-axis.

Another description: Rotate the line $y=\frac{r}{h}\cdot x$, $0\leq x\leq h$, about the x-axis.

[3] (b) Use the formula for the volume of a solid of revolution to express the volume of the right-circular cone with base radius r and height h as a definite integral. **DO NOT EVALUATE THE INTEGRAL.**

$$\text{Volume} = \int_0^h \pi\left(\frac{r}{h}\cdot x\right)^2 dx$$

[8] **6.** Evaluate $\int_2^3 \frac{2x^3}{\sqrt{x^2-1}}\,dx$. Hint: Write $2x^3 = 2x \cdot x^2$.

We use the Substitution Rule.

$u = x^2 - 1 \implies x^2 = u + 1$ (*)

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$$\int_2^3 \frac{2x^3}{\sqrt{x^2-1}}\,dx = \int_2^3 \frac{2x \cdot x^2}{\sqrt{x^2-1}}\,dx =$$

$$= \int_3^8 \frac{x^2}{\sqrt{u}}\,du \overset{(*)}{=} \int_3^8 \frac{u+1}{\sqrt{u}}\,du =$$

$$= \int_3^8 \left(\sqrt{u} + \frac{1}{\sqrt{u}}\right) du = \left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_3^8 =$$

$$= \left(\frac{16}{3}\sqrt{8} + 2\sqrt{8}\right) - \left(\frac{6}{3}\sqrt{3} + 2\sqrt{3}\right) = \boxed{\frac{22}{3}\sqrt{8} - 4\sqrt{3}}$$

$$\approx 13.8136$$